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FINAL REPORT

A DISTRIBUTION MODEL FOR THE AERIAL APPLICATION OF GRANULAR AGRICULTURAL PARTICLES

by

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Abstract

A model is developed to predict the shape of the distribution of granular agricultural particles applied by aircraft. The particle is assumed to have a random size and shape and the model includes the effect of air resistance, distributor geometry and aircraft wake. General requirements for the maintenance of similarity of the distribution for scale model tests are derived and are addressed to the problem of a non-general drag law. It is shown that if the mean and variance of the particle diameter and density are scaled according to the scaling laws governing the system, the shape of the distribution will be preserved. Distributions are calculated numerically and show the effect of a random initial lateral position, particle size and drag coefficient. A listing of the computer code is included.

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NOMENCLATURE

b	wing semispan (m)
C_D	particle drag coefficient
C_L	aircraft lift coefficient
d	particle diameter (m)
g	acceleration due to gravity (m/sec^2)
g^*	non-dimensional acceleration due to gravity
K	constant in drag relation
K_v	constant in initial velocity relation
m	particle mass (kg)
t	time (sec)
U	aircraft speed (m/sec)
u	non-dimensional x-velocity of particle
u_a	x-velocity of air (m/sec)
u_p	particle x-velocity (m/sec)
v	non-dimensional y-velocity of particle
v_a	y-velocity of air (m/sec)
v_o	initial y-velocity of particle
v_p	particle y-velocity (m/sec)
V_r	magnitude of relative particle-air velocity (m/sec)
w	non-dimensional z-velocity of particle
w_a	z-velocity of air (m/sec)
w_o	initial z-velocity of particle
w_p	particle z-velocity (m/sec)
x	longitudinal particle coordinate
y	lateral particle coordinate

y_G	lateral ground intersection point
y_o	initial lateral coordinate of particle
z	vertical particle coordinate
z_o	initial vertical coordinate of particle
α	modified ballistic parameter
β	ballistic parameter
δ	non-dimensional particle diameter
η	non-dimensional y-velocity of air
μ	mean (expected) value
ν	kinematic viscosity of air (m^2/sec)
ζ	non-dimensional z-velocity of air
ρ	non-dimensional density of particle
ρ_a	air density (gm/cm^3)
ρ_p	particle density (gm/cm^3)
σ	square root of variance (standard deviation)
τ	non-dimensional time

I. INTRODUCTION

Aerial application of material has played an increasingly significant role in United States agriculture, forestry, and other related industries since the Second World War. One of the major problems associated with the use of aircraft in this field has been the difficulty of obtaining desired distribution patterns with present equipment. This problem has been addressed by several groups in the academic and research fields, notably Henry (1962) and Yates, et al. (1970). However, most of this work has been of a trial and error nature with only sketchy guidelines being offered by theoretical considerations, mostly with regard to particle trajectories.

It would appear, then, that an investigation of the factors which go to make up the structure of the distribution and which directly affect its shape would be of some use in designing better aircraft/distributor systems. Accordingly, this paper presents a model of the distribution of dry agricultural material which is based on the application of basic probability theory. The model takes into account the random nature of the size and shape of the particles, air resistance, distributor geometry and the effect of aircraft wake. Some results, based on data obtained for three grains (wheat, corn and oats), are presented in order to demonstrate the applicability and to show the effect of certain parameters on the distribution. In addition, a discussion of the effects of scaling on the distribution is presented and requirements for maintaining similarity in the distribution are discussed.

II. PARTICLE CHARACTERISTICS

One of the first requirements in the development of a method for predicting the distributions of agricultural particles is a knowledge of the physical and aerodynamic characteristics of these particles. A literature search revealed that the necessary data were available for some particles but would have to be collected from numerous individual sources and investigations. An attempt was made to gather as much information as possible on these characteristics from the literature. No attempt was made to make additional measurements.

It was found that fairly complete characteristics could be assembled for several of the grains (corn, oats and wheat), while the same information was almost totally lacking for all of the solid fertilizers. The data presented here were collected from references (3-9), and were reduced via the method outlined below.

The delineation of physical characteristics which best suited the point of view of this study was to assume the particle to have three fundamental characteristics which define its physical properties. They are: density, size (which together determine the weight) and shape (which determines the drag coefficient relation). A sample collection of one type of particle will contain specimens possessing a wide range of each of these characteristics. That is, the characteristics will possess an apparent random nature, even though each individual specimen will have a fixed density and size, and will possess a definite relation between its drag coefficient and Reynolds number. The form of this relation is dependent on the shape of the particle (just as a streamlined body possesses a different drag coefficient from a flat plate or a sphere) and thus

allowing the drag coefficient to be a random variable will reflect the random nature of the particle shape.

Here the drag coefficient is defined in the conventional way

$$C_D = \frac{D}{\frac{1}{2}\rho_a V_r^2 S}$$

where V_r is the magnitude of the particle velocity relative to the air around it and S is a characteristic area associated with the particle, commonly taken to be its frontal area. As stated before, the drag coefficient is a function of the Reynolds number of the particle, given as

$$Re = \frac{V_r d}{\nu}$$

where d is a characteristic linear dimension of the particle. Above, ρ_a and ν are the density and kinematic viscosity of the air, respectively.

With this approach, then, the actual particle shape and its random nature is important only to the extent that it affects the drag coefficient relation and hence is completely independent of the size of the particle. Therefore, any convenient and consistent method for sizing the particles is acceptable. The approach taken here is one proposed by Garrett and Brooker (1965). The particle size is determined by considering a sphere which has the same volume as the particle in question and using the corresponding diameter of that sphere as the characteristic linear dimension. The characteristic area then is simply the cross-sectional area of the same sphere. The drag coefficient, measured by any of several methods described in the literature (the most common being a measurement of the terminal velocity of the particle), then corresponds to a Reynolds number calculated

using the diameter of an equal volume sphere. Thus, a drag relation can be constructed by a statistical analysis of the data.

A least squares method was used to fit a general relation of the form

$$C_D = A\ Re + B/Re + C \tag{1}$$

to the data and the results are shown in Figures 1-3. This relation was taken to be the mean, or expected value, for the drag relation. A log-normal distribution function was used to describe the random variation of C_D about this mean. This distribution has two primary properties: First, if a random variable X is lognormal, then ln X is normal, or gaussian. Second, the function goes to zero as X approaches zero and is undefined for X less than zero. We should not expect any particles to have drag coefficients less than zero.

In addition, this distribution, like the gaussian distribution, is completely characterized by its mean value and its variance. The small amount and large scatter of the data for the variance makes any attempt at curve fitting questionable, however a least squares fit of a cubic

$$\sigma_{C_D} = D\ Re^3 + E\ Re^2 + F\ Re + G \tag{2}$$

was made. The results are presented in Figures 4-6.

The particle size, as determined by the diameter of an equal volume sphere, was also held to be described by a lognormal distribution. However, the analysis was of a more straightforward nature and the results are

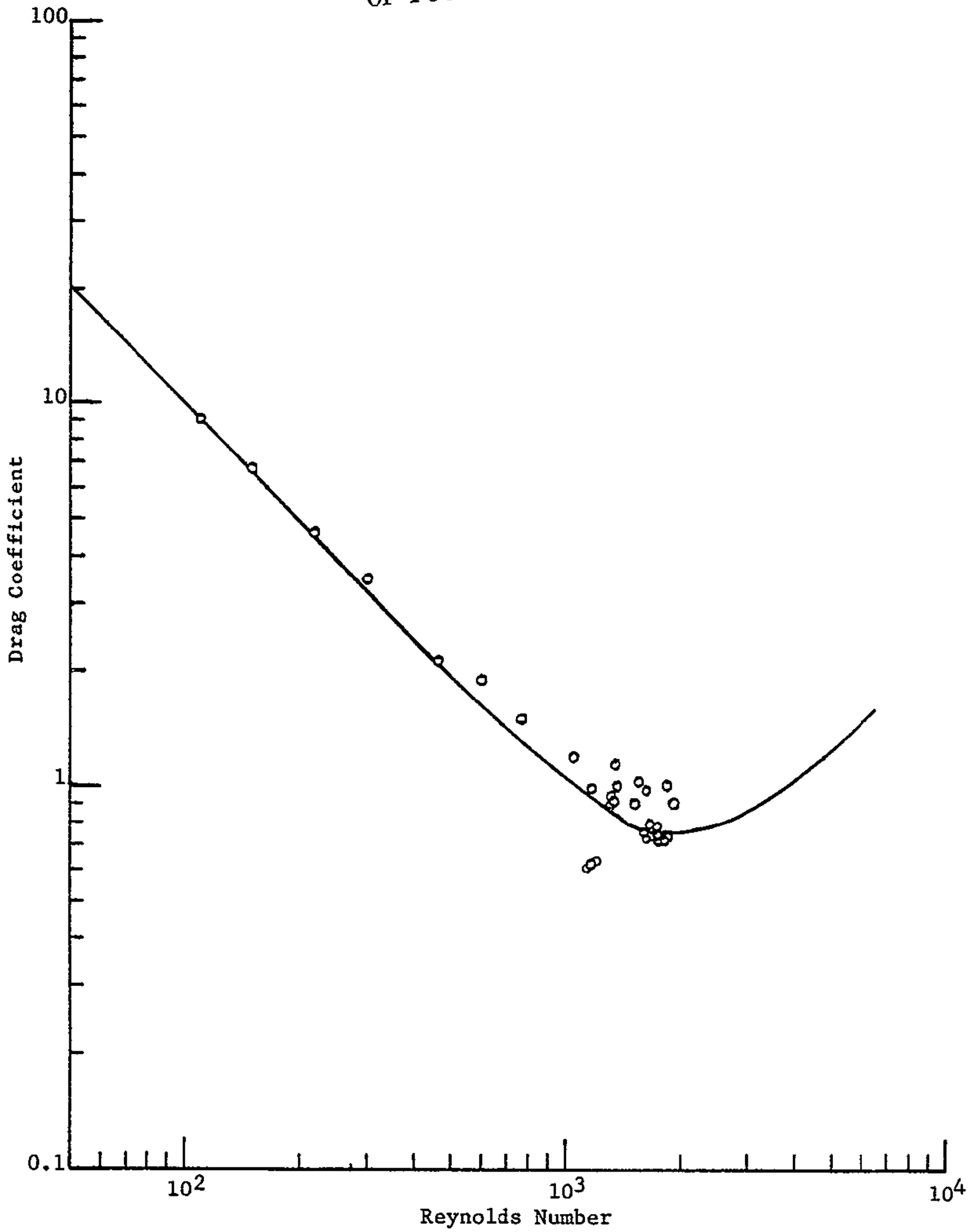

Figure 1. Drag Relation for Wheat

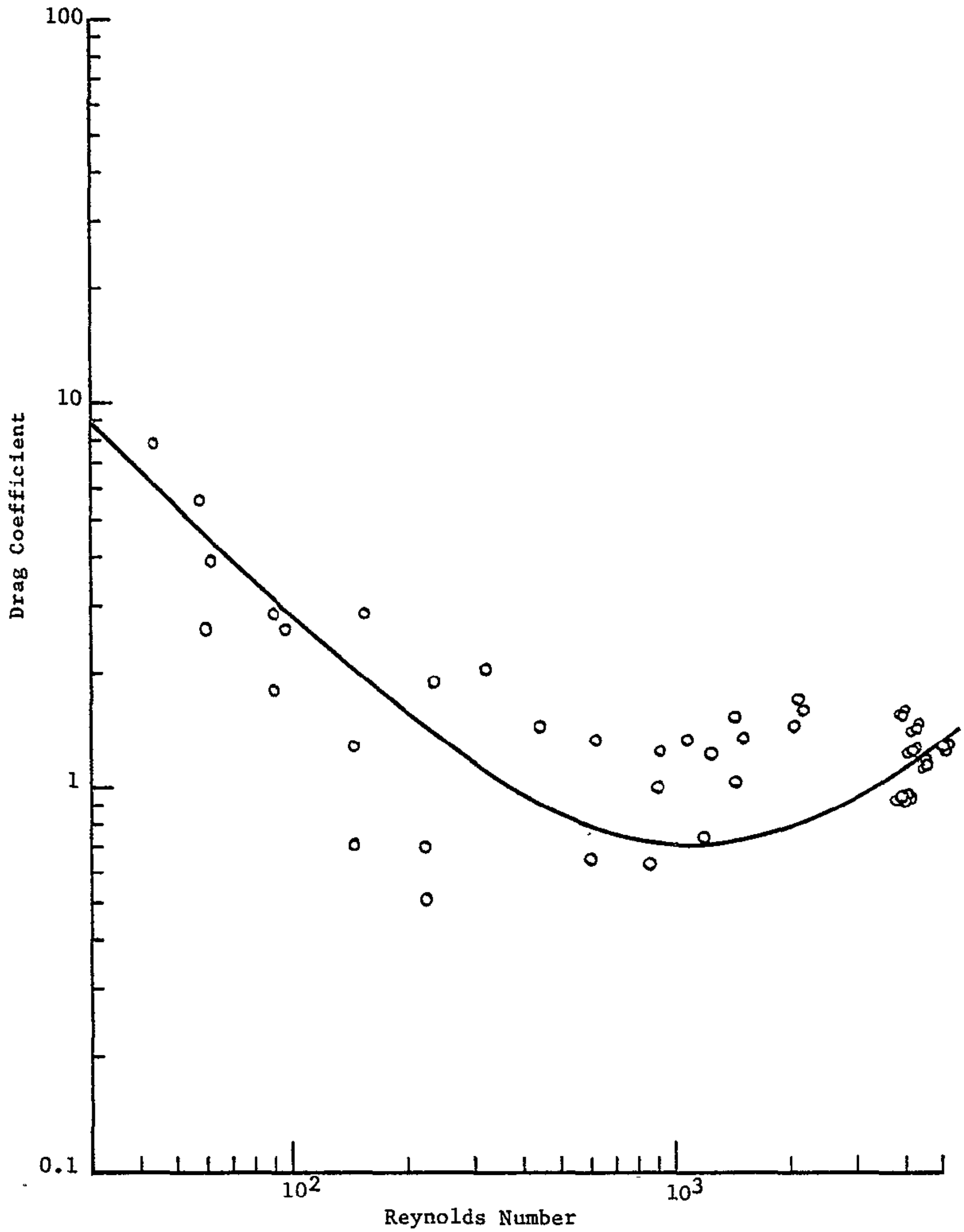

Figure 2. Drag Relation for Corn

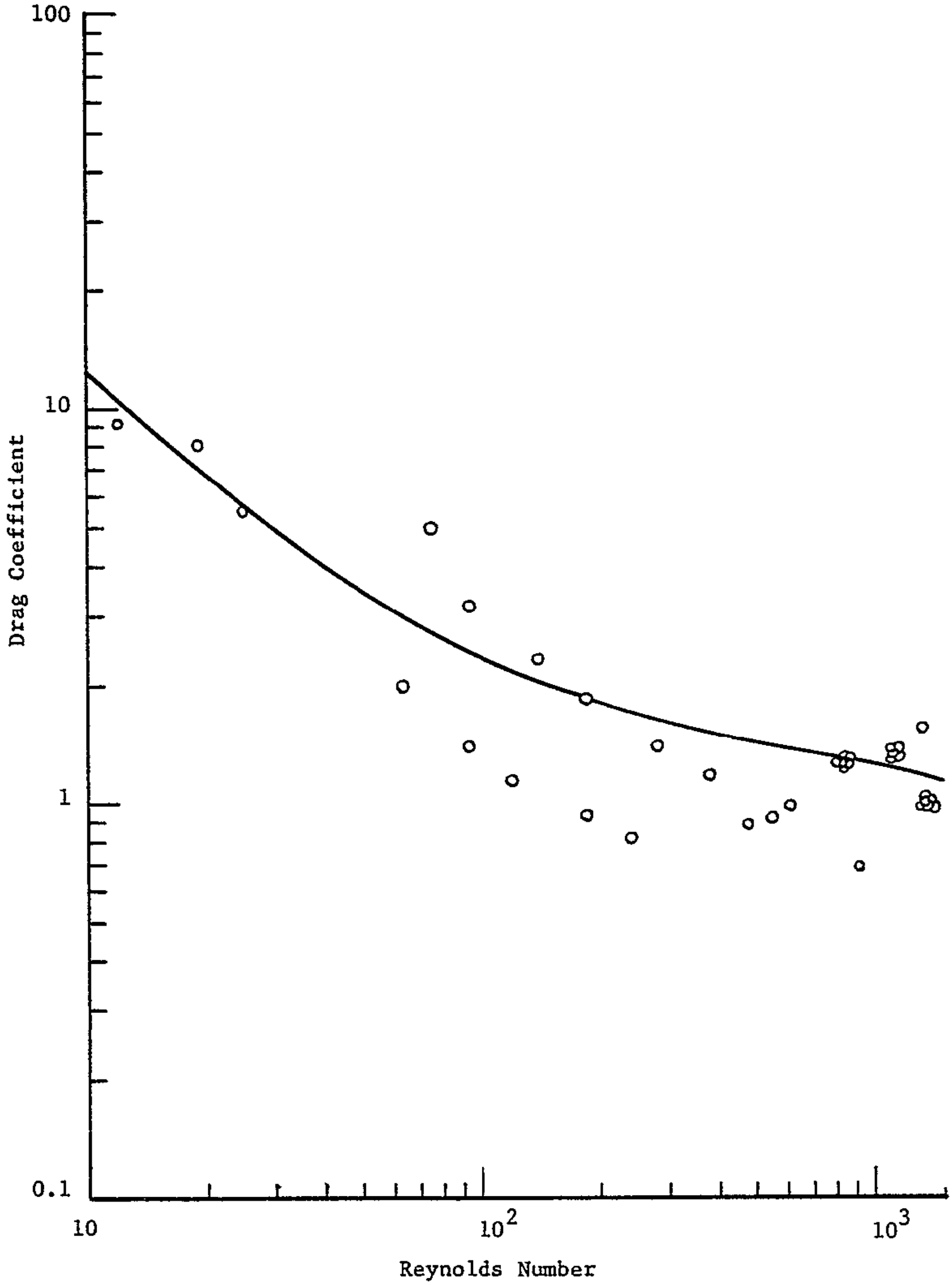

Figure 3. Drag Relation for Oats

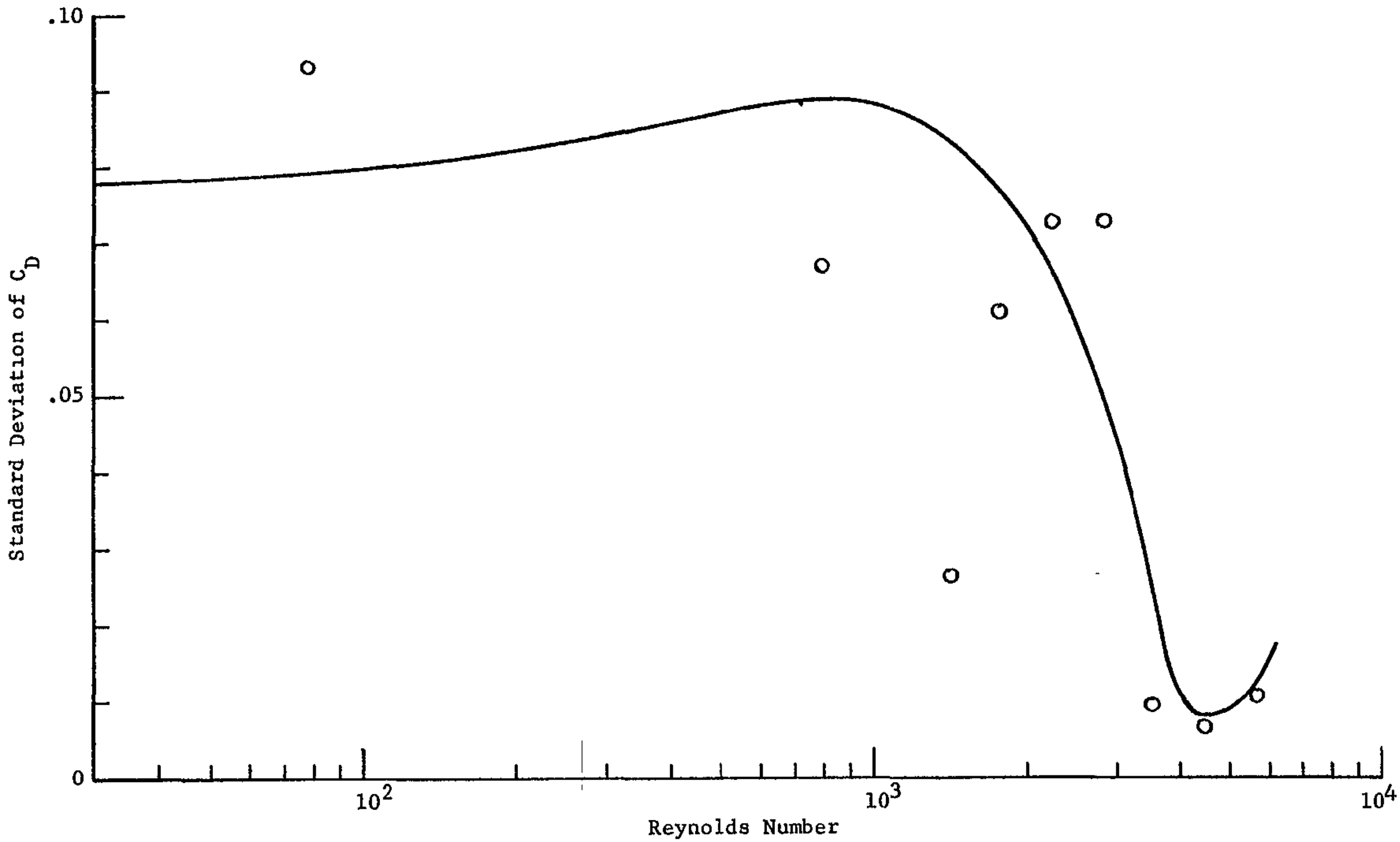

Figure 4. Standard Deviation for Wheat

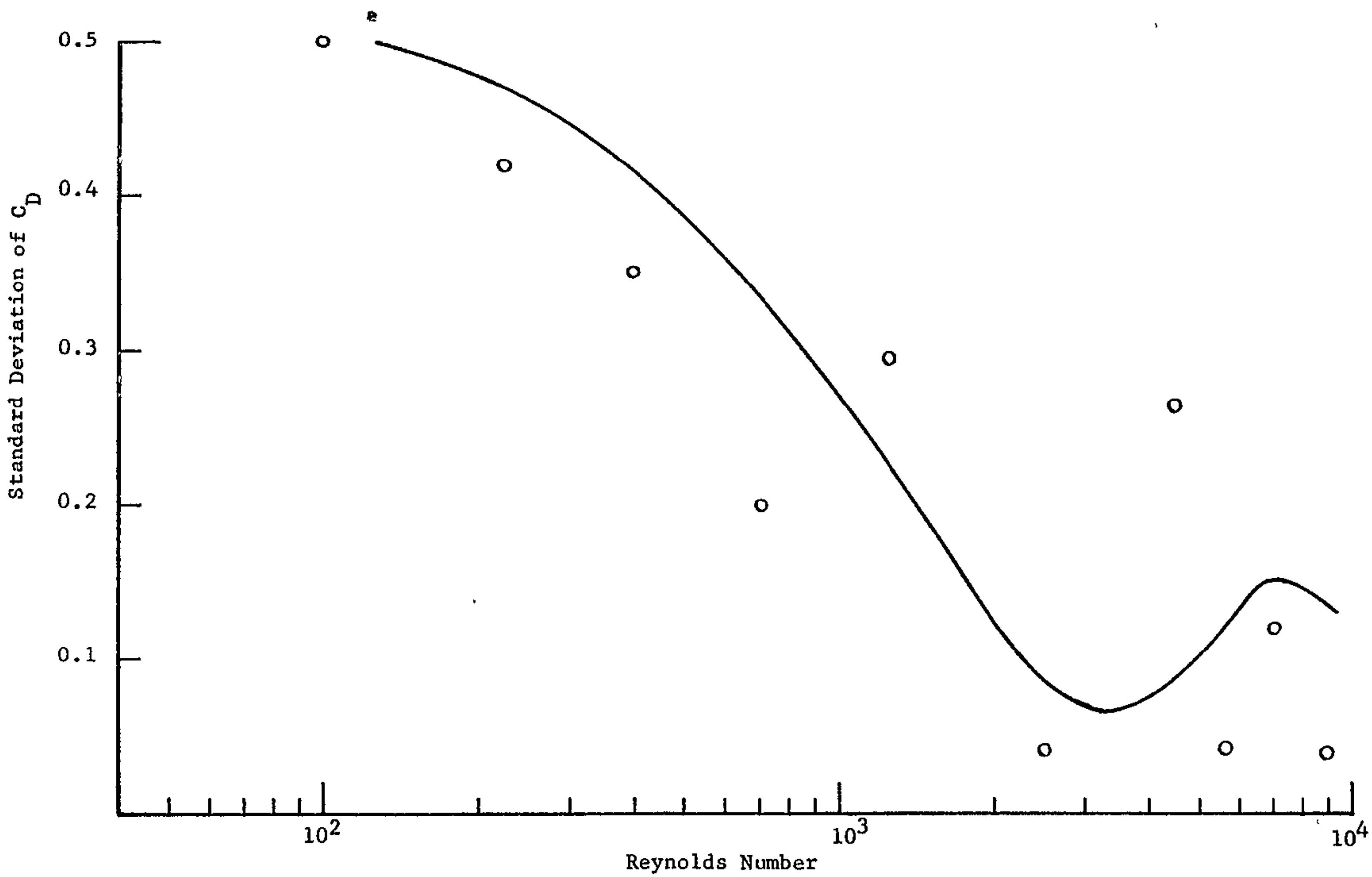

Figure 5. Standard Deviation for Corn

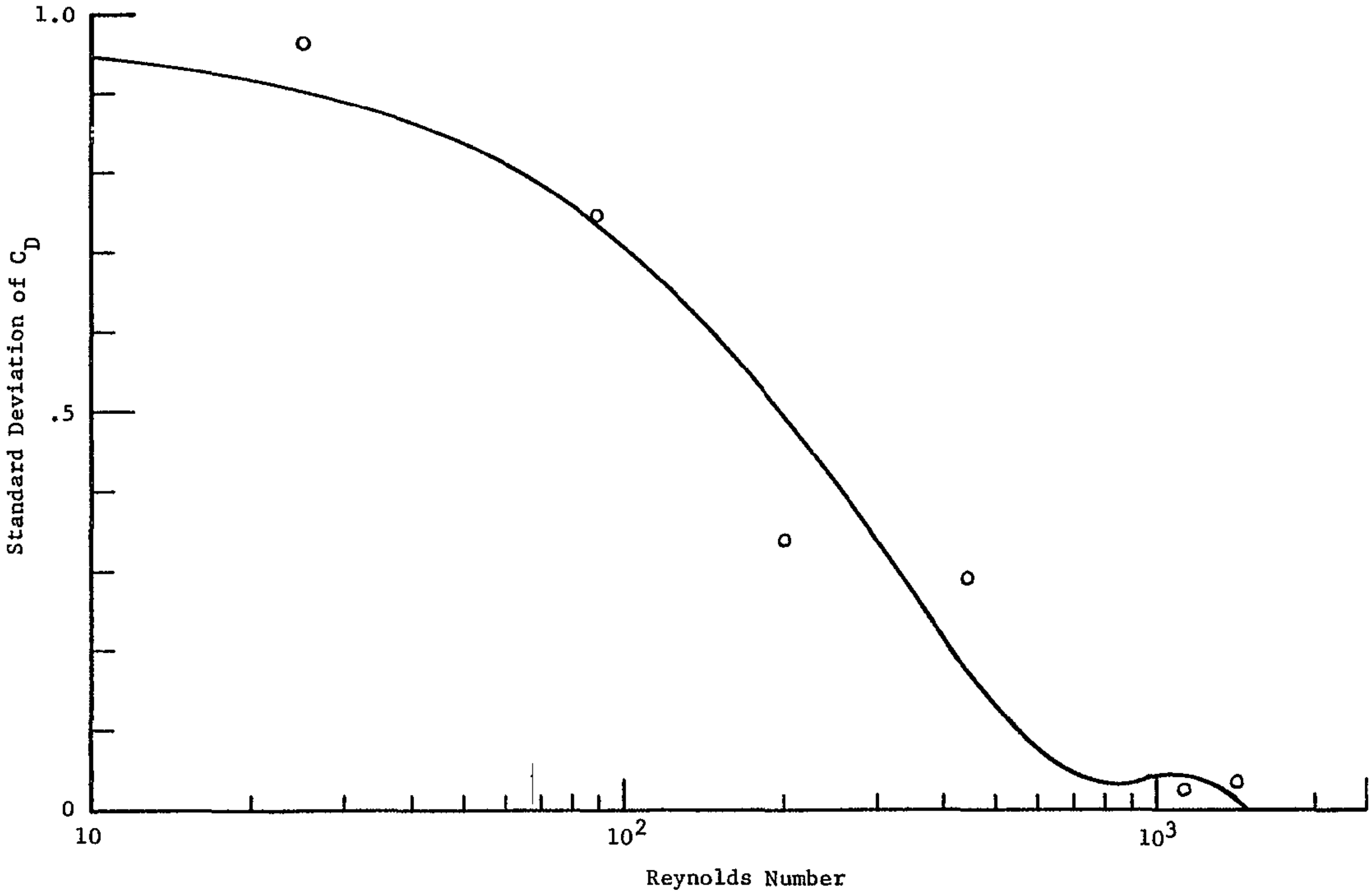

Figure 6. Standard Deviation for Oats

tabulated in Table 1. It was found that the density of these particles did not vary significantly from specimen to specimen (probably for biological reasons) and so it is safe to assume the density to be a deterministic quantity. The values used are also presented in Table 1.

In the calculation of the standard deviation of the drag coefficient, each point on the graph represents at least six data points, as it was necessary to group neighboring points on the drag curve in order to obtain a standard deviation for a particular range of Reynolds number.

Table 1

Physical Characteristics

	Diameter (microns)		
	Mean	Standard Deviation	Density (g/cm^3)
Oats	3594	426	1.369
Corn	8133	338	1.234
Wheat	3573	189	1.320

III. TRAJECTORY EQUATIONS

Each particle, as it is ejected from the aircraft, becomes a free projectile and is subject to Newton's second law of motion. The equation of motion, in vector form, describing the trajectory is

$$m \frac{d\vec{V}_p}{dt} + \vec{D} + \vec{L} + m\vec{g} = 0 \qquad (3)$$

where L refers to the lift force generated by the particle. Since the particles studied here are irregular in nature (in particular, oats have a flattened profile), they will, in general, produce a force perpendicular to the direction of motion, as well as one opposite the direction of motion. These forces are termed lift and drag, respectively. Though, as shown in the preceding section, there is information regarding the drag experienced by these particles, predicting and representing the lift forces is a more complex matter. It is felt that, by ignoring the lift force, the subsequent simplification will be of such a degree as to offset the relatively small loss of accuracy.

With this simplification, then, the single vector equation can be written more revealingly as three scalar equations, with the component of the drag force in the $i^{\underline{th}}$ direction being

$$D_i = |\vec{D}| \frac{U_{ip} - U_{ia}}{|\vec{V}_r|}$$

so that the drag acts in the opposite direction to the velocity vector

$$m \frac{du_p}{dt} + D_x = 0 \qquad (4)$$

$$m \frac{dv_p}{dt} + D_y = 0 \tag{5}$$

$$m \frac{dw_p}{dt} + D_z - mg = 0 \tag{6}$$

It is apparent, however, from the work of Bragg (1977), that the motion of the particle remains predominantly in a plane perpendicular to the direction of motion of the aircraft (the x-direction) and since the distribution is dependent only on the motion in this plane, a two dimensional model will be used for this study.

Rewriting the y and z equations of motion with the three fundamental particle characteristics explicitly shown, gives, after dividing through by the particle mass

$$\frac{dv_p}{dt} + \frac{3}{4} \frac{\rho_a}{\rho_p} \frac{(v_p - v_a)}{d} C_D \sqrt{(v_p - v_a)^2 + (w_p - w_a)^2} = 0 \tag{7}$$

$$\frac{dw_p}{dt} - g + \frac{3}{4} \frac{\rho_a}{\rho_p} \frac{(w_p - w_a)}{d} C_D \sqrt{(v_p - v_a)^2 + (w_p - w_a)^2} = 0 \; . \tag{8}$$

Since the wake system of the aircraft is dependent primarily on the aircraft geometry and velocity, the nondimensionalization was carried out with respect to the aircraft flight speed U and the wing semi-span, b. The equations then become

$$\frac{dv}{d\tau} + \frac{3}{4} \frac{\rho_a}{\rho_p} \frac{(v-\eta)}{\delta} C_D \sqrt{(v-\eta)^2 + (w-\zeta)^2} = 0 \tag{9}$$

$$\frac{dw}{d\tau} - g^* + \frac{3}{4} \frac{\rho_a}{\rho_p} \frac{(w-\zeta)}{\delta} C_D \sqrt{(v-\eta)^2 + (w-\zeta)^2} = 0 \tag{10}$$

where the nondimensional variables are defined in Table 2.

Table 2

Nondimensionalization

v, w	$=$	$\frac{v_p}{U}$, $\frac{w_p}{U}$
η, ζ	$=$	$\frac{v_a}{U}$, $\frac{w_a}{U}$
τ	$=$	$\frac{tU}{b}$
δ	$=$	$\frac{d}{b}$
g^*	$=$	$\frac{gb}{U^2}$

Detailed numerical studies of the above system have been carried out by Reed (1953) and Bragg with respect to liquid droplets. Although the trajectories predicted in both cases seemed to indicate good agreement with experiment, these analyses could only yield distributions by the time consuming and costly method of a numerical compilation of trajectories calculated using a matrix of initial conditions and particle parameters. It seemed desirable to construct an approximate solution to the above system which would offer a simple method of constructing the distributions while preserving some of the accuracy of the previous analyses.

Returning to equations (9) and (10), one of the most striking aspects revealed there is the fact that the particle-dependent parameters can be grouped into a single variable, and it is this variable alone which affects the solution of the system. This parameter bears a close resemblance to the ballistic coefficient, which is used in the study of projectile motion, and so it can be referred to as the ballistic parameter, β, where

$$\beta = \frac{3}{4}\frac{\rho_a}{\rho_p}\frac{C_D}{\delta} \quad .$$

Because of the presence of the drag coefficient, the ballistic parameter is a function of the particle Reynolds number. However, extensive simplification of the entire system can be achieved by observing that a simple drag relation of the form

$$C_D = K/Re \tag{11}$$

offers nearly as good a fit to the data as equation (1).

Substitution and further manipulation with this result yields the following system:

$$\frac{dv}{d\tau} + \alpha(v-\eta) = 0 \tag{12}$$

$$\frac{dw}{d\tau} - g^* + \alpha(w-\zeta) = 0 \tag{13}$$

where a modified form of the ballistic parameter, α, has been used

$$\alpha = \frac{3}{4}\,\frac{\nu}{Ub}\,\frac{\rho_a}{\rho_p}\,\frac{K}{\delta^2} = c\,\frac{K}{\delta^2} \quad . \tag{14}$$

Although it appears that much simplification has been achieved, this system still requires numerical solution due to the complex nature of the dependence of η and ζ on position.

The range of values assumed by α for the grains studied in this report necessitated the use of two approximate solutions to this system. The first is to consider α to be vanishingly small. This corresponds to the case where the particle is quite large and dense, or the medium through which it travels is quite rarified and the drag coefficient low. This results in the classical case of a body in free fall in a constant gravitational field. The solution for the trajectory is simply

$$y = y_o + v_o\tau \tag{15}$$

$$z = z_o + w_o\tau - g^*\frac{\tau^2}{2} \quad . \tag{16}$$

The lateral position at which the particle strikes the ground can be calculated by letting $z = 0$ and solving equation (16) for τ, and substituting into equation (15). The result is

$$y_G = y_o + \frac{v_o}{g^*}\,(w_o + \sqrt{w_o^2 + zz_o g^*}) \quad . \tag{17}$$

Comparison of this result with a numerical integration of equation (12-13) showed excellent agreement for values of α up through about .20 corresponding to a diameter of about 3500 microns, as shown in Figure 7.

The second approximation was arrived at by considering initially the case of a particle falling through still air, that is, neglecting the aircraft wake. A study of this system revealed that the smaller particles (larger α) tended to be carried further outboard and also have a significantly lower flight time in the numerical computation than in this approximation. It is apparent that the effect of the wake behind the aircraft is to draw the particles downward and outward from the aircraft fuselage. This effect is primarily due to the pair of vortices which are shed from each wing tip as a result of the lifting action of the wing and which cause the air in the vicinity of the vortices to rotate in the manner shown in Figure 8.

In accordance with these observations, a modification to the still-air model was made which extended the range of validity to cover the grains being studied. This modification consists of adding a constant air velocity whose magnitude and direction is consistent with those found in an aircraft wake and which result in the best agreement with the numerical calculation. The adjusted equations then become

$$\frac{dv}{d\tau} + \alpha v - \alpha\eta = 0 \tag{18}$$

$$\frac{dw}{d\tau} - g^* + \alpha w - \alpha\zeta = 0 \tag{19}$$

where η and ζ are constants. The solution for the trajectory is simply

$$y = y_o + \eta\tau + \frac{v_o - \eta}{\alpha}\,(1-e^{-\alpha\tau}) \tag{20}$$

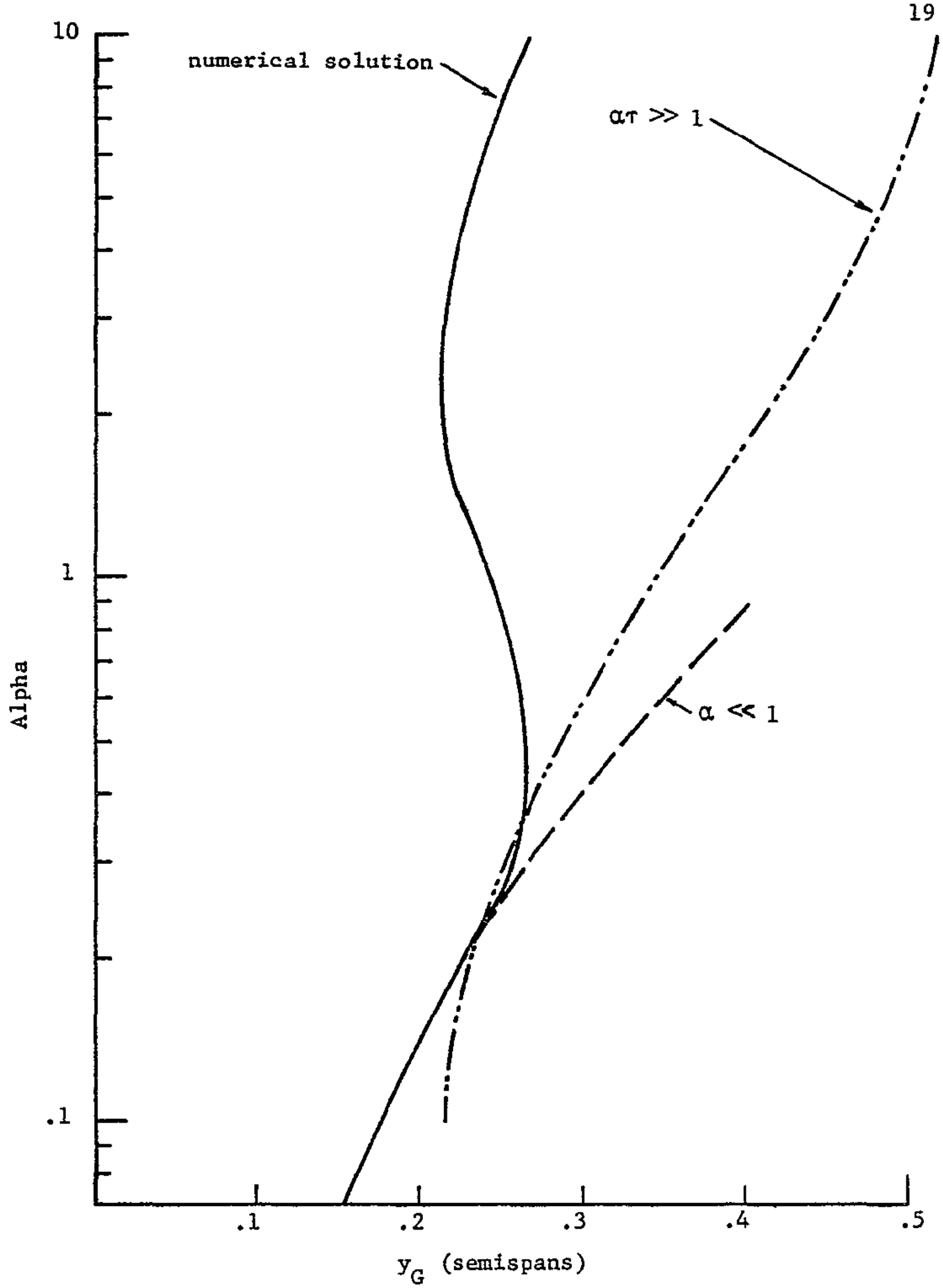

Figure 7. Approximate Solutions

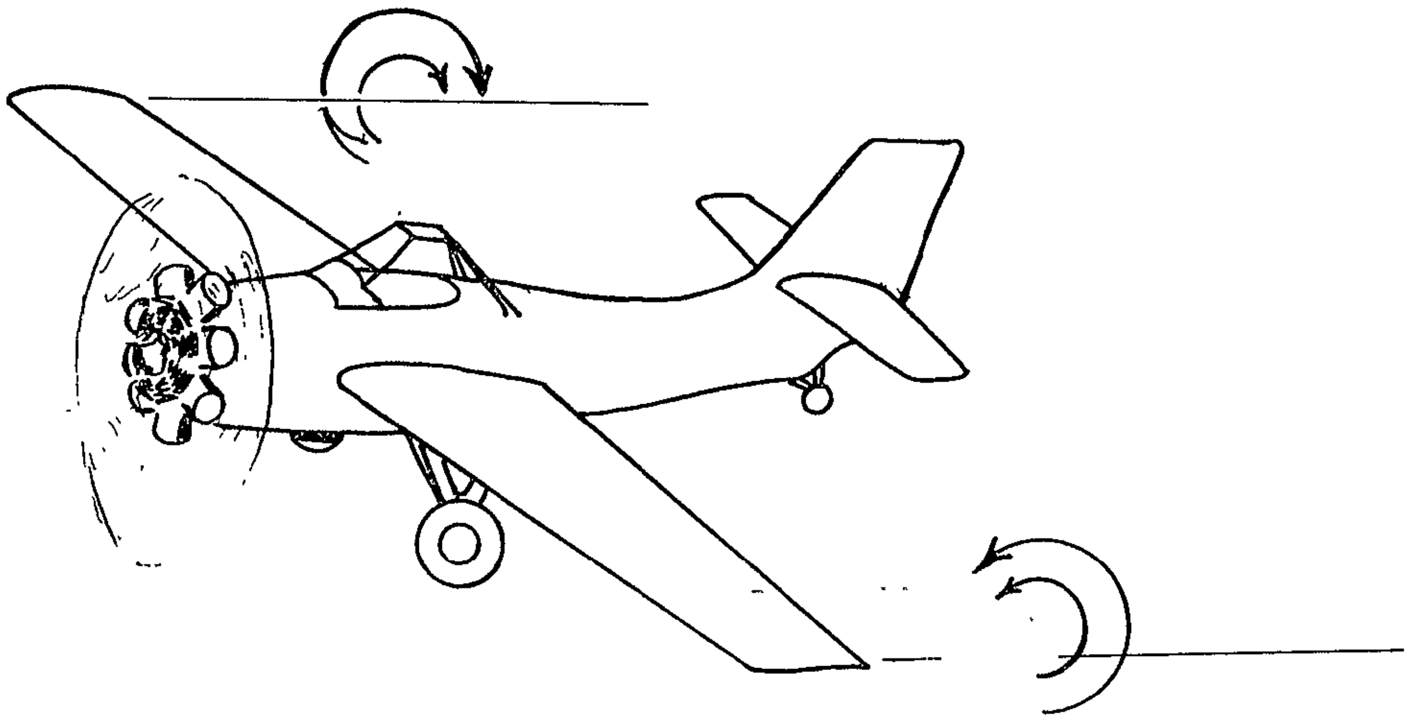

Figure 8. Wake Structure

$$z = z_o + (\zeta - \frac{g^*}{\alpha})\tau + \frac{w_o - \zeta + \frac{g^*}{\alpha}}{\alpha}(1-e^{-\alpha\tau}) \ . \qquad (21)$$

Sample comparisons between the trajectories calculated with equations (15-16) and (20-21) and those arrived at numerically are shown in Figures 9-14. Since the cases with larger α had longer flight times, an expression for the lateral position of the ground intersection can be derived by assuming

$$e^{-\alpha\tau_G} \approx 0$$

resulting in the expression

$$y_G = y_o + \frac{v_o}{\alpha} + \frac{\eta}{\alpha}\left(\frac{z_o\alpha + w_o}{\frac{g^*}{\alpha} - \zeta}\right) \ . \qquad (22)$$

Comparison of this result with the numerical calculation is also shown in Figure 7. It will be seen later that this extensive approximation is necessary in order to conduct the probability analysis.

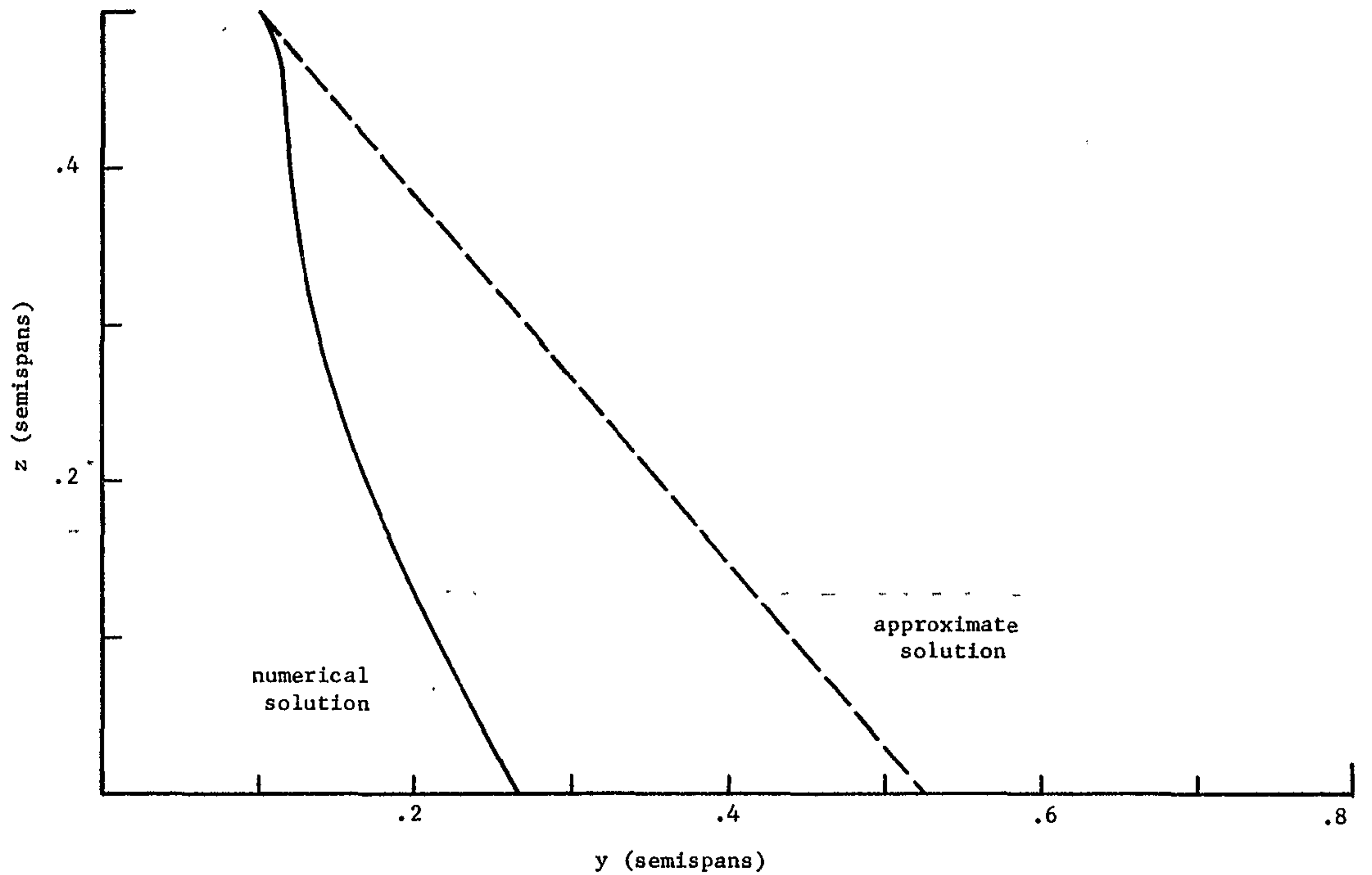

Figure 9. Trajectories for $\alpha = 9.810$ ($d \approx 500$ microns)

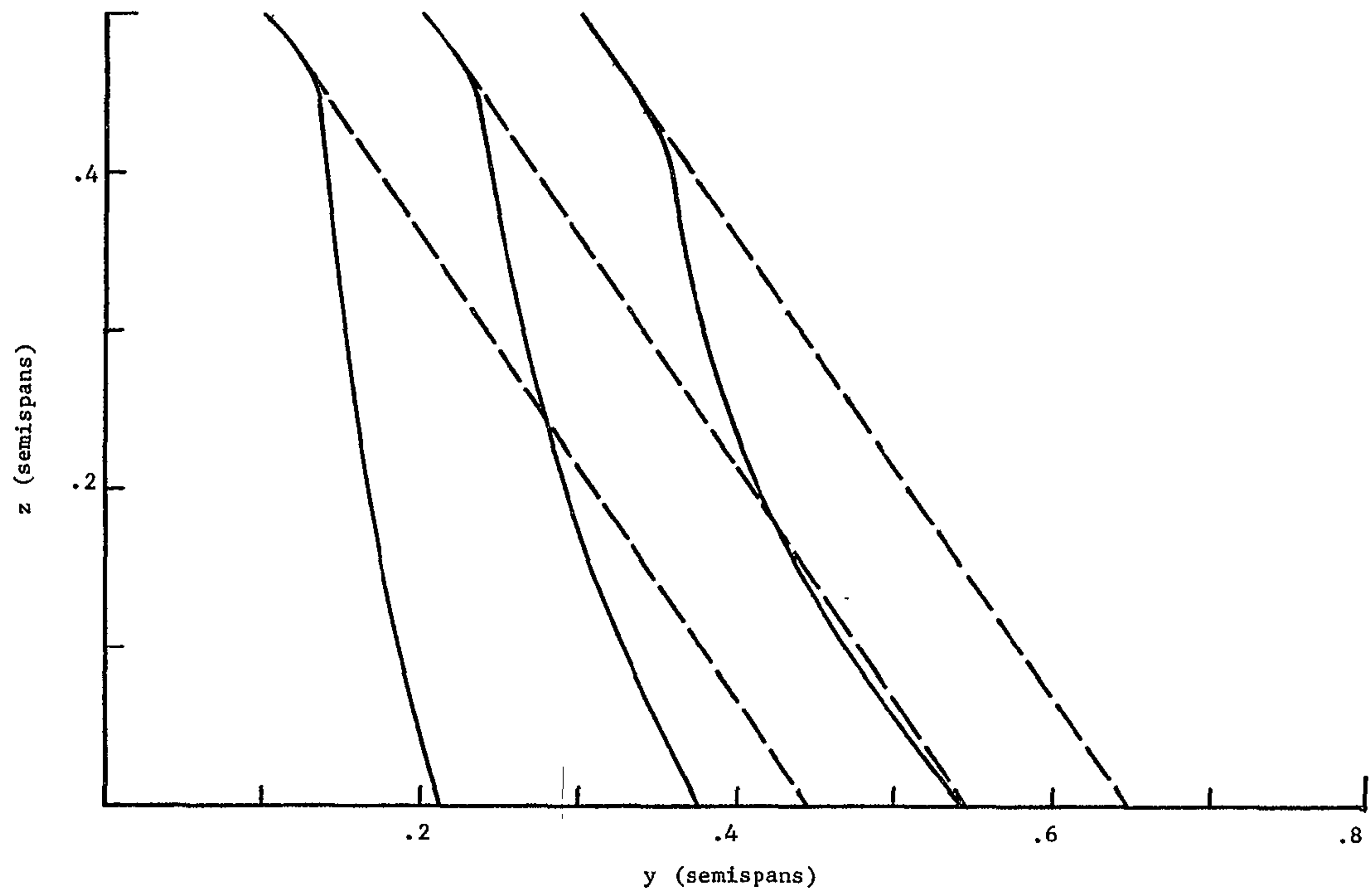

Figure 10. Trajectories for $\alpha = 2.453$ ($d \approx 1000$ microns)

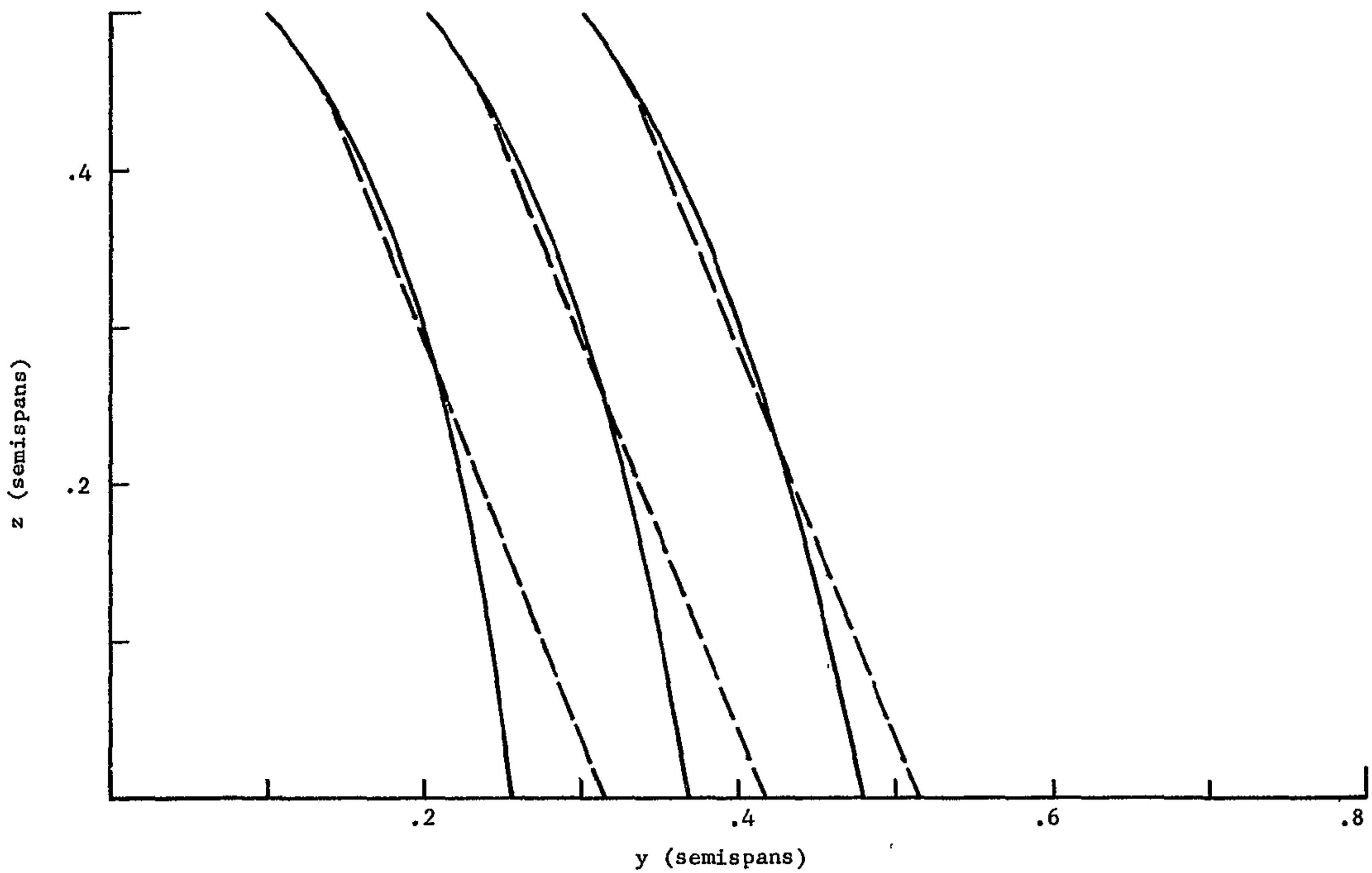

Figure 11. Trajectories for $\alpha = 0.613$ ($d \approx 2000$ microns)

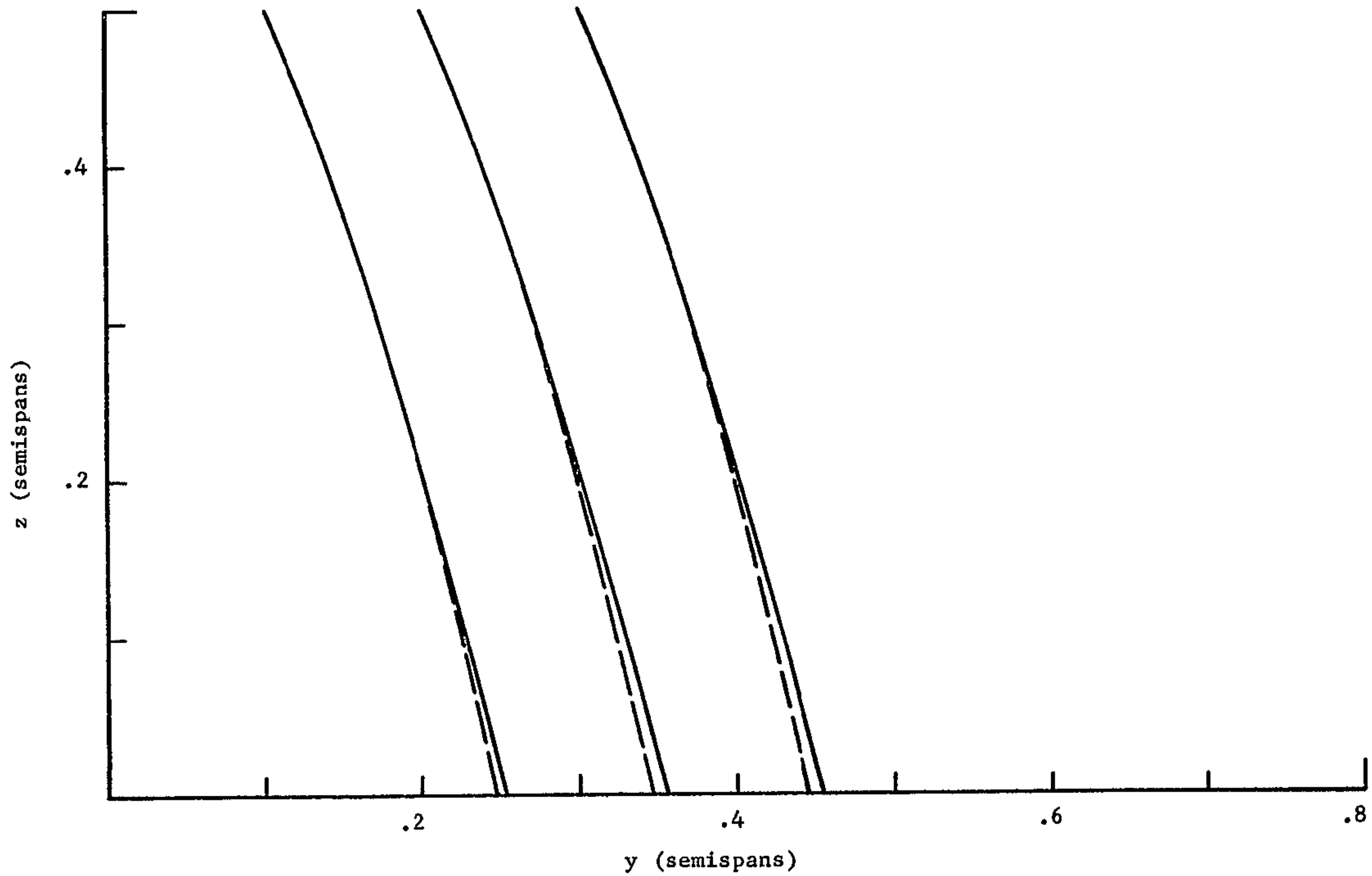

Figure 12. Trajectories for $\alpha = 0.273$ ($d \approx 3000$ microns)

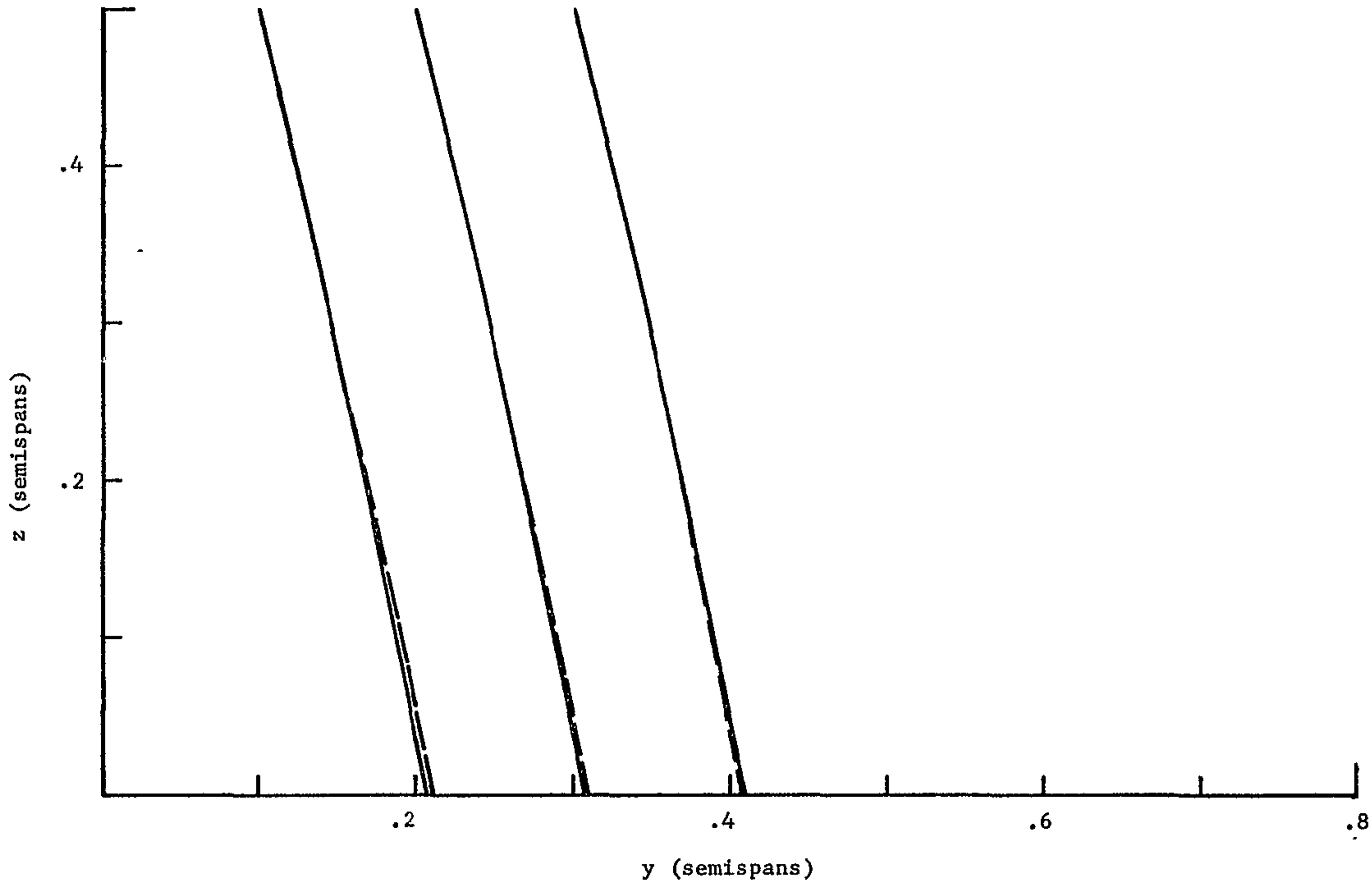

Figure 13. Trajectories for $\alpha = 0.153$ ($d \approx 4000$ microns)

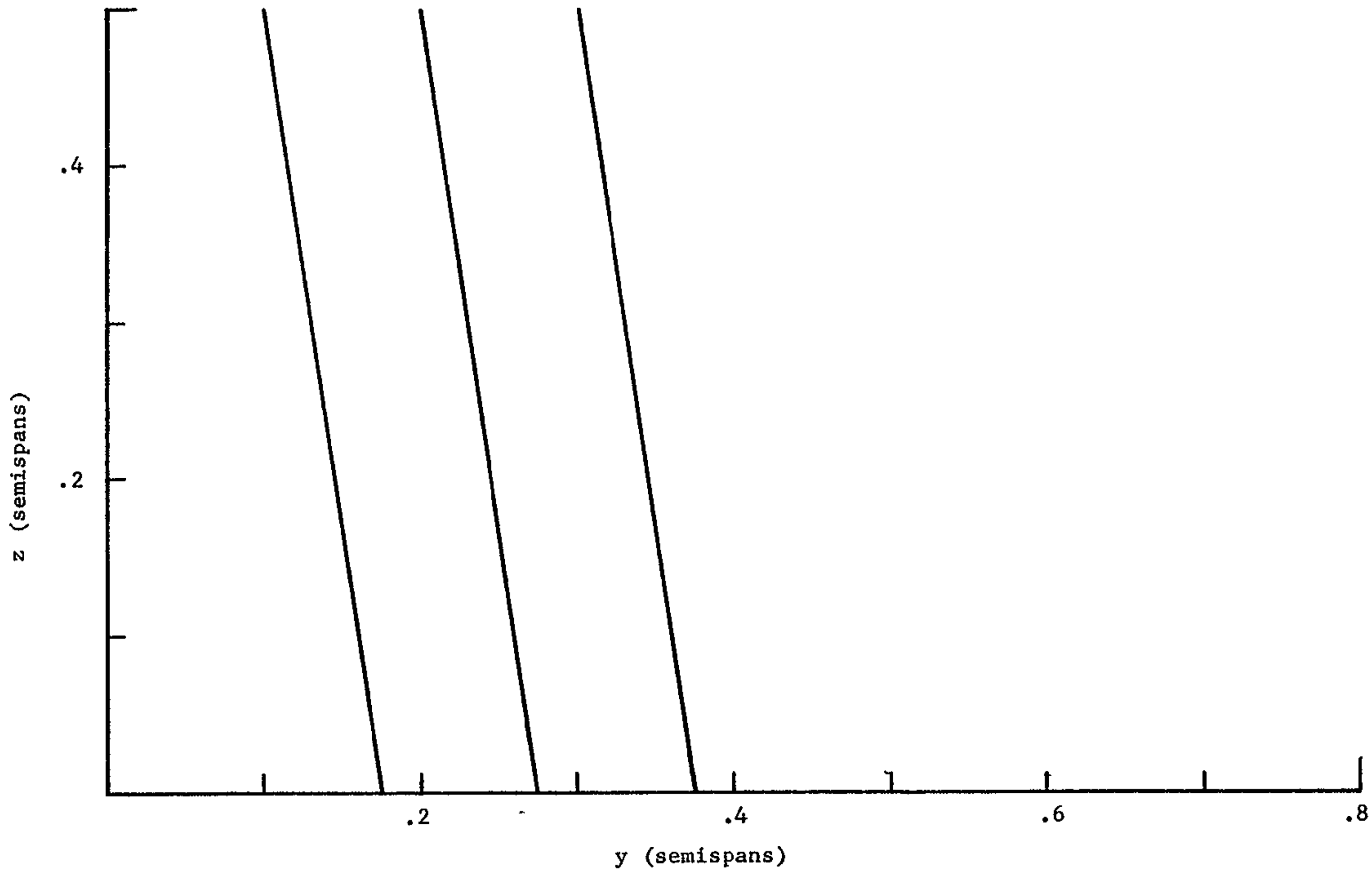

Figure 14. Trajectories for α = 0.098 (d $\approx$ 5000 microns)

IV. THE DISTRIBUTION MODEL

As stated earlier, this model makes use of results from basic probability theory.

In general, the ground intersection point, y_G, of a particle which is ejected from an aircraft is a function of the initial conditions of its trajectory, its particle parameters, the aircraft geometry and flight conditions. For the system defined here, with the appropriate non-dimensionalizations, this can be written as

$$y_G = F(y_o, z_o, v_o, w_o, \alpha, C_L) \tag{23}$$

where C_L, the aircraft lift coefficient, is a measure of the magnitude of the wake velocities induced by the tip vortices. In the sense that these variables can assume a range of possible values for any given type of particle and aircraft/distributor system, they can be thought of as random variables. As such, they will possess a joint probability density function (PDF) characterizing their random nature of the form

$$P_{Y_o, Z_o, V_o, W_o, A, C_L}(y_o, z_o, v_o, w_o, \alpha, C_L) \ .$$

This is the most general representation of the probabilistic system in that the individual PDF's can be derived from it by first fixing the values of the other variables, then multiplying by the probability that each will assume that fixed value, and finally integrating over all possible values of the fixed variables, so that, for instance,

$$p_{Y_o}(y_o) = \int_{z_o} \cdots \int_{v_o w_o \alpha C_L} p_{Y_o|Z_o,V_o,W_o,A,C_L}(y_o|z_o,v_o,w_o,\alpha,C_L)$$

$$\times\, p_{Z_o,V_o,W_o,A,C_L}(z_o,v_o,w_o,\alpha,C_L)\,dz_o dv_o dw_o d\alpha dC_L$$

or more simply

$$p_{Y_o}(y_o) = \int_{z_o} \cdots \int_{v_o w_o \alpha C_L} p_{Y_o,Z_o,W_o,A,C_L}(y_o,z_o,v_o,w_o,\alpha,C_L)$$

$$\times\, dz_o dv_o dw_o d\alpha dC_L \; .$$

Further, since it is known that if a function of a random variable defines a one to one mapping onto another random variable, the PDF for the second variable is defined as

$$p_Y(y) = p_X(x)\left|\frac{dx}{dy}\right|$$

so that by again fixing all the variables in equation (23), an expression for the PDF of y_G can be derived. Writing first

$$p_{Y_G|Z_o,V_o,W_o,A,C_L}(y_G|z_o,v_o,w_o,\alpha,C_L)$$

$$= p_{Y_o|Z_o,V_o,W_o,A,C_L}(y_o|z_o,v_o,w_o,\alpha,C_L) \cdot \left|\frac{dy_o}{dy_G}\right|$$

then multiplying through and integrating as before, the result is

$$p_{Y_G}(y_G) = \int_{z_o,} \cdots \int_{v_o,w_o,\alpha,C_L} p_{Y_o,Z_o,V_o,W_o,A,C_L}(y_o,z_o,v_o,w_o,\alpha,C_L)$$

$$\times \left|\frac{dy_o}{dy_G}\right| dz_o dv_o dw_o d\alpha dC_L \; . \qquad (24)$$

Considering that many particles go to make up a single deposition, the probability density function will give a good representation of the actual deposition shape. Note also that the same result can be arrived at by fixing any combination of the variables and integrating over those. In addition, since it can be assumed that the particle ballistic parameter is independent of the initial conditions, that is, the particle characteristics do not depend on the distributor geometry, the joint PDF can be split and equation (24) can be rewritten as

$$p_{Y_G}(y_G) = \int\limits_{z_o, v_o, w_o, \alpha, C_L} \cdots \int p_{Y_o, Z_o, V_o, W_o, C_L}(y_o, z_o, v_o, w_o, C_L) \times p_A(\alpha) \left|\frac{dy_o}{dy_G}\right| dz_o dv_o dw_o d\alpha dC_L \; . \tag{25}$$

This, then, is the general equation describing the ground distribution. The necessity of an analytical expression for y_G in terms of the other parameters is apparent due to the presence of the term dy_o/dy_G.

V. SOME DISTRIBUTIONS

In the interest of demonstrating the applicability of the model derived in this study, only two parameters were considered to have a random nature, y_o and α. The other parameters were represented as functions of these, or as constants. Extension to the more general case is straightforward and merely requires additional integration. Specifically, z_o and C_L were held as constants, and v_o and w_o are represented by

$$v_o = K_v y_o$$

$$w_o = w_o(\alpha) \; .$$

The resulting simplified expression for the PDF of y_G is

$$p_{Y_G}(y_G) = \int_\alpha p_{Y_o}(y_o) p_A(\alpha) \left|\frac{dy_o}{dy_G}\right| d\alpha \tag{26}$$

where

$$\left|\frac{dy_o}{dy_G}\right| = \begin{cases} \dfrac{1}{1 + \dfrac{Kv}{g}\left(w_o + \sqrt{w_o^2 + 2z_o g^*}\right)} & 0 \leq \alpha \leq .20 \\[2ex] \dfrac{1}{1 + \dfrac{Kv}{\alpha}} & \alpha > .20 \end{cases} \tag{27}$$

and the PDF for α can be derived as follows. Equation (14) can be rewritten as

$$\ln \alpha = \ln C + \ln K - 2 \ln \delta \; . \tag{28}$$

A basic result of probability theory is that the sum of any number of gaussian random variables is also gaussian. It can then be concluded

that, since ln K and ln δ are gaussian, ln α is gaussian. The effect of the constant is merely to shift the mean. Finally, since ln α is gaussian, α is lognormal. Its mean and variance are computed as follows.

Let

$$Z = \ln \alpha$$

$$Y = \ln K$$

$$X = \ln \delta$$

$$C^* = \ln C \quad .$$

Then

$$Z = C^* + Y - 2X$$

and the expected value of Z, or the mean, written as E[Z], is simply

$$E[Z] = C^* + E[Y] - 2E[Z] \; ,$$

a more common notation for the mean is μ, so that

$$\mu_Z = C^* + \mu_Y - 2\mu_X \; . \tag{29}$$

The variance, written as $E[(Z-\mu_Z)^2]$ is simply

$$E[(Z-\mu_Z)^2] = \sigma_Z^2$$

or

$$E[(Z^2-2Z\mu_Z+\mu_Z{}^2)] = E[Z^2] - 2E[Z]\mu_Z + \mu_Z{}^2 = E[Z^2] - \mu_Z{}^2 \; .$$

Substituting for Z and expanding as above

$$\sigma_Z{}^2 = E[Y^2] + 4E[X^2] + C^{*2} + 2C^*E[Y] - 4C^*E[X] - 4E[Y]E[X] - \mu_Z{}^2 \; .$$

Further substitution gives

$$\sigma_Z^{\ 2} = \sigma_Y^{\ 2} + 4\sigma_X^{\ 2} + (C* - 2\mu_X + \mu_Y)^2 - \mu_Z^{\ 2}$$

or

$$\sigma_Z^{\ 2} = \sigma_Y^{\ 2} + 4\sigma_X^{\ 2} . \tag{30}$$

The general formula for the lognormal PDF is

$$p_t(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp[\frac{(\ln t - \mu)^2}{2\sigma^2}] \qquad t \geq 0 \tag{31}$$

where the values for σ and μ are those used above. The actual expected value of t is given by

$$E[t] = \int_0^\infty t \cdot p_t(t)dt$$

which results in

$$\mu_t = E[t] = e^{\mu + \frac{\sigma^2}{2}} , \tag{32}$$

and the variance of t, $\sigma_t^{\ 2}$ is given by

$$\sigma_t^{\ 2} = E[(t-\mu_t)^2] = \int_0^\infty t^2 p_t(t)dt - \mu_t^{\ 2}$$

and the result is

$$\sigma_t^{\ 2} = \mu_t^{\ 2}(e^{\sigma^2} - 1) . \tag{33}$$

Equation (32) and (33) furnish relations between the actual mean and standard deviations of a random variable t (μ_t and σ_t) and those values used in the lognormal PDF (μ and σ). With these results, then, the PDF for α can be written as

$$p_\alpha(\alpha) = \frac{1}{\sigma\alpha\sqrt{2\pi}} \exp[\frac{(\ln \alpha - \mu)^2}{2\sigma^2}] \tag{34}$$

where

$$\mu = \ln C + \mu_1 - 2\mu_2$$

$$\sigma^2 = \sigma_1{}^2 + 4\sigma_2{}^2$$

and

$$\sigma_1{}^2 = \ln[1 + (\frac{\sigma_K}{\mu_K})^2]$$

$$\sigma_2{}^2 = \ln[1 + (\frac{\sigma_\delta}{\mu_\delta})^2]$$

$$\mu_1 = \ln \mu_K - \frac{\sigma_1{}^2}{2}$$

$$\mu_2 = \ln \mu_\delta - \frac{\sigma_2{}^2}{2} \quad .$$

A computer program was written to numerically integrate equation (26), applying the above results. It is called DEP and a listing is provided in the Appendix. The variation of y_o is provided in the form of FUNCTION subroutines PYOF and PYOG which can be easily modified to account for any type of variation of y_o. For the case in which y_o is a constant, that is, there is a single orifice or nozzle, the resulting distribution for wheat is shown in Figure 15.

For the case in which the PDF of y_o is constant across a given width, results are shown in Figures 16-20 in which the type of material, then the width of the distributor is varied.

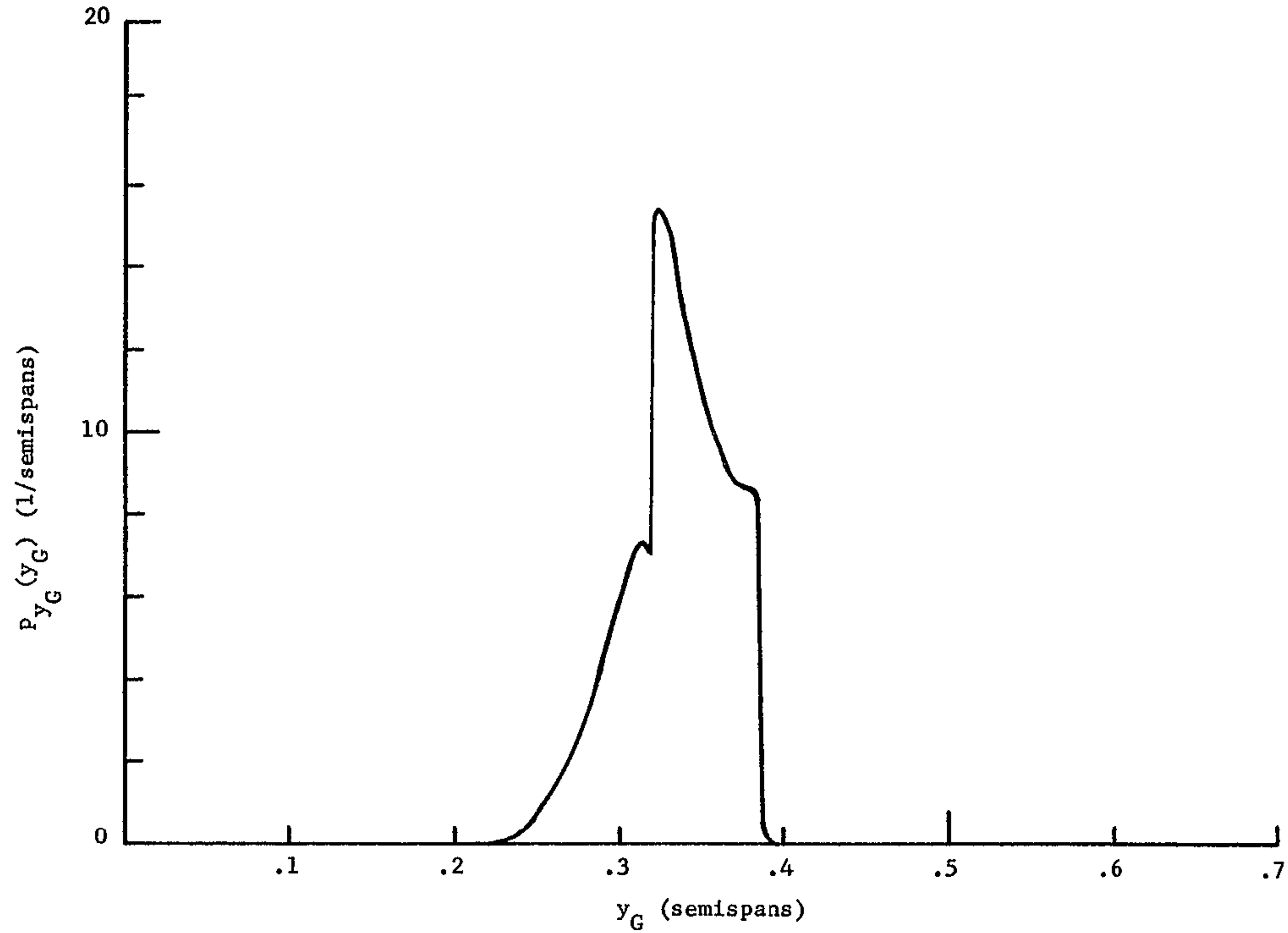

Figure 15. Wheat, $.05 \le y_o \le .06$

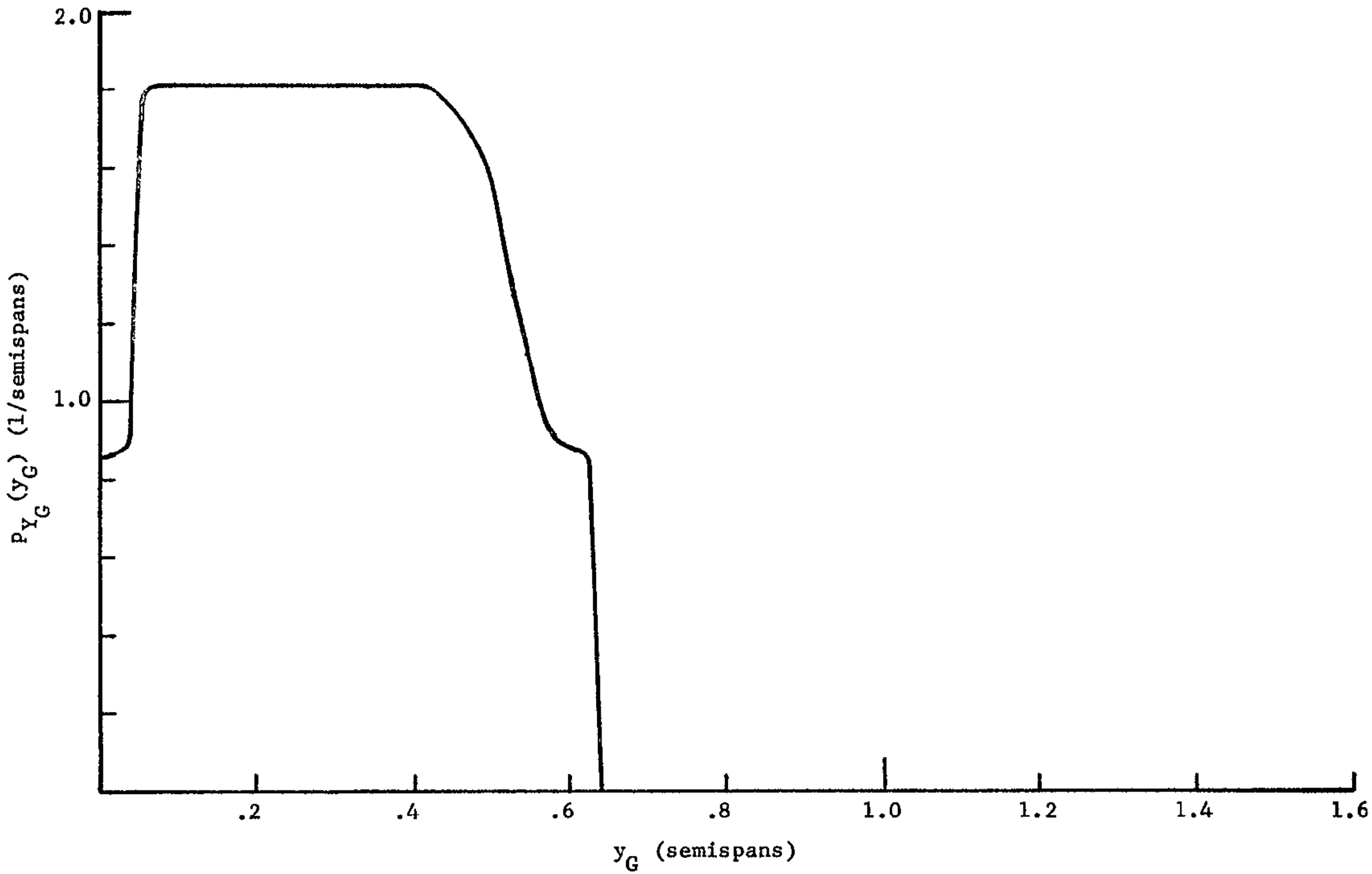

Figure 16. Wheat, $0 \leq y_o \leq 0.1$

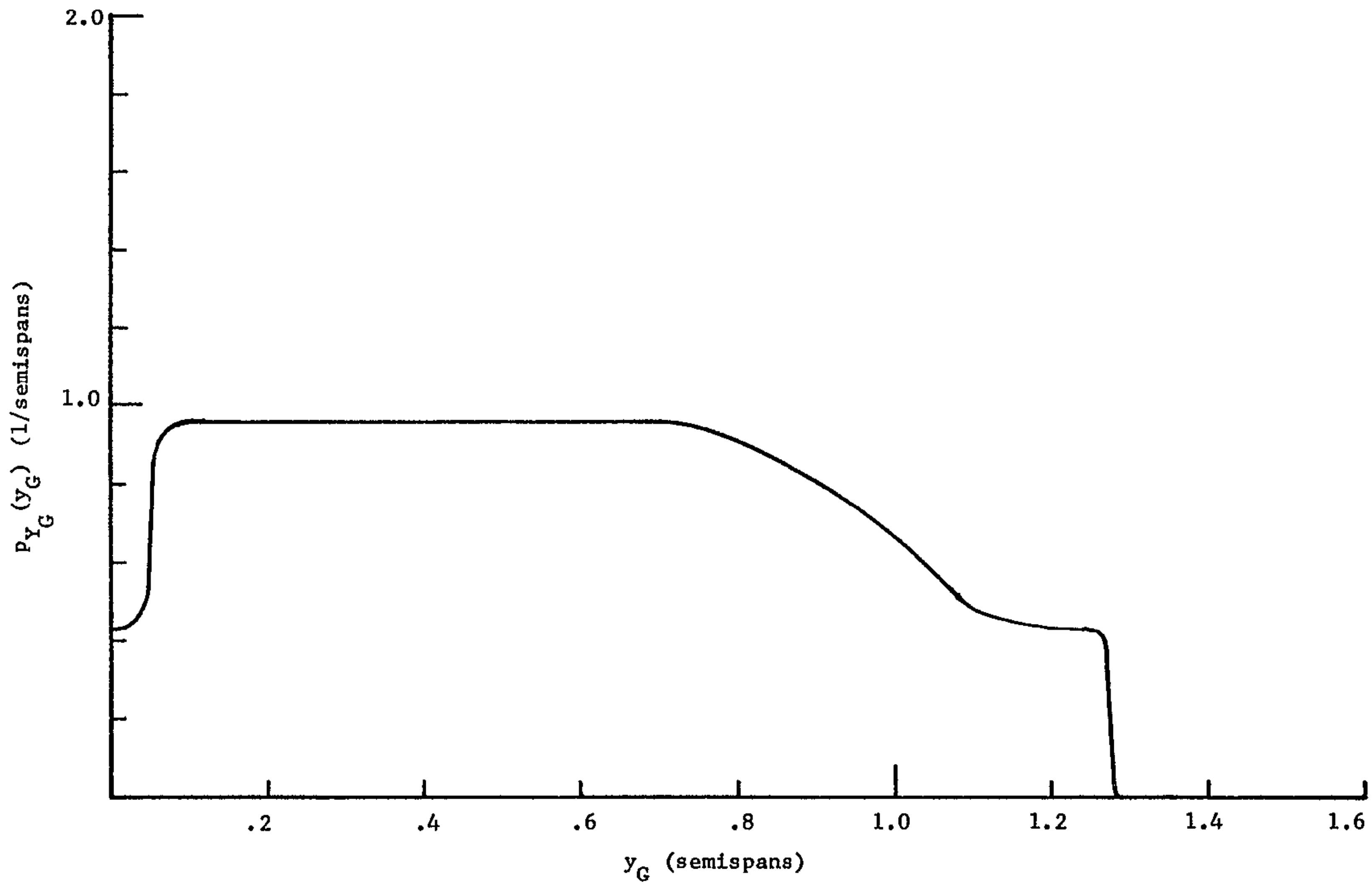

Figure 17. Wheat, $0 \leq y_o \leq 0.2$

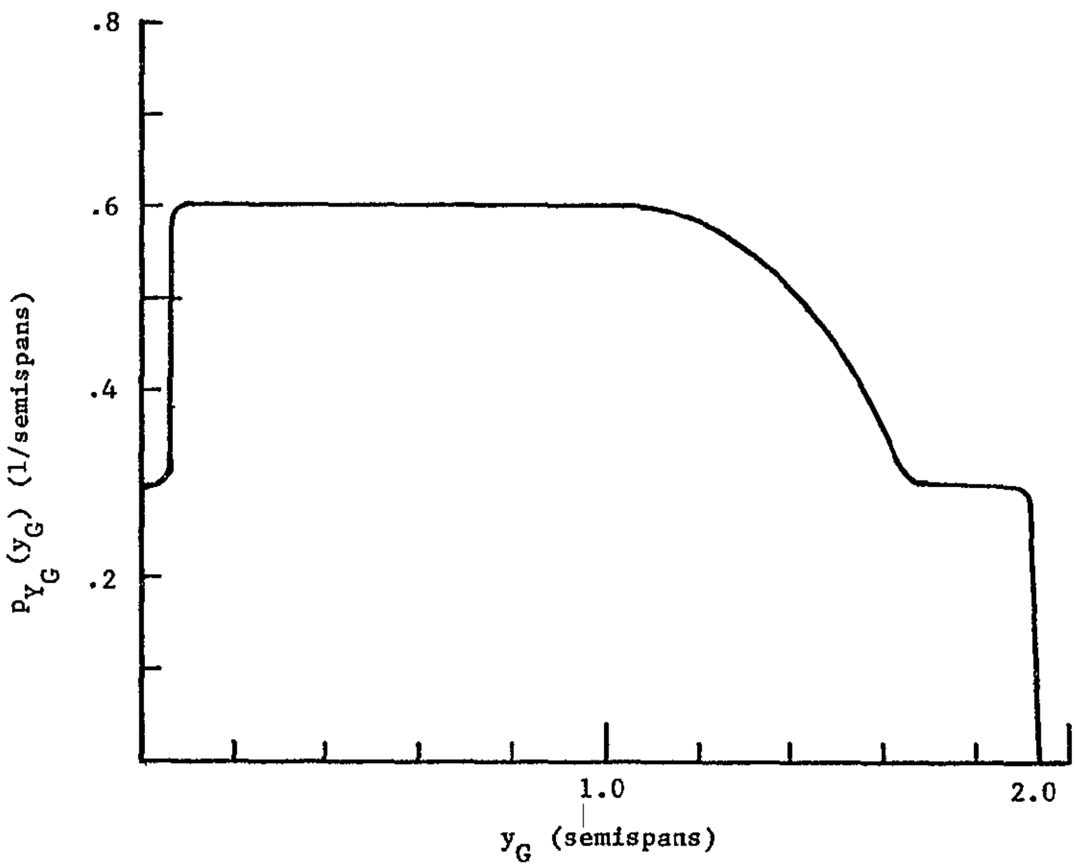

Figure 18. Wheat, $0 \leq y_o \leq 0.3$

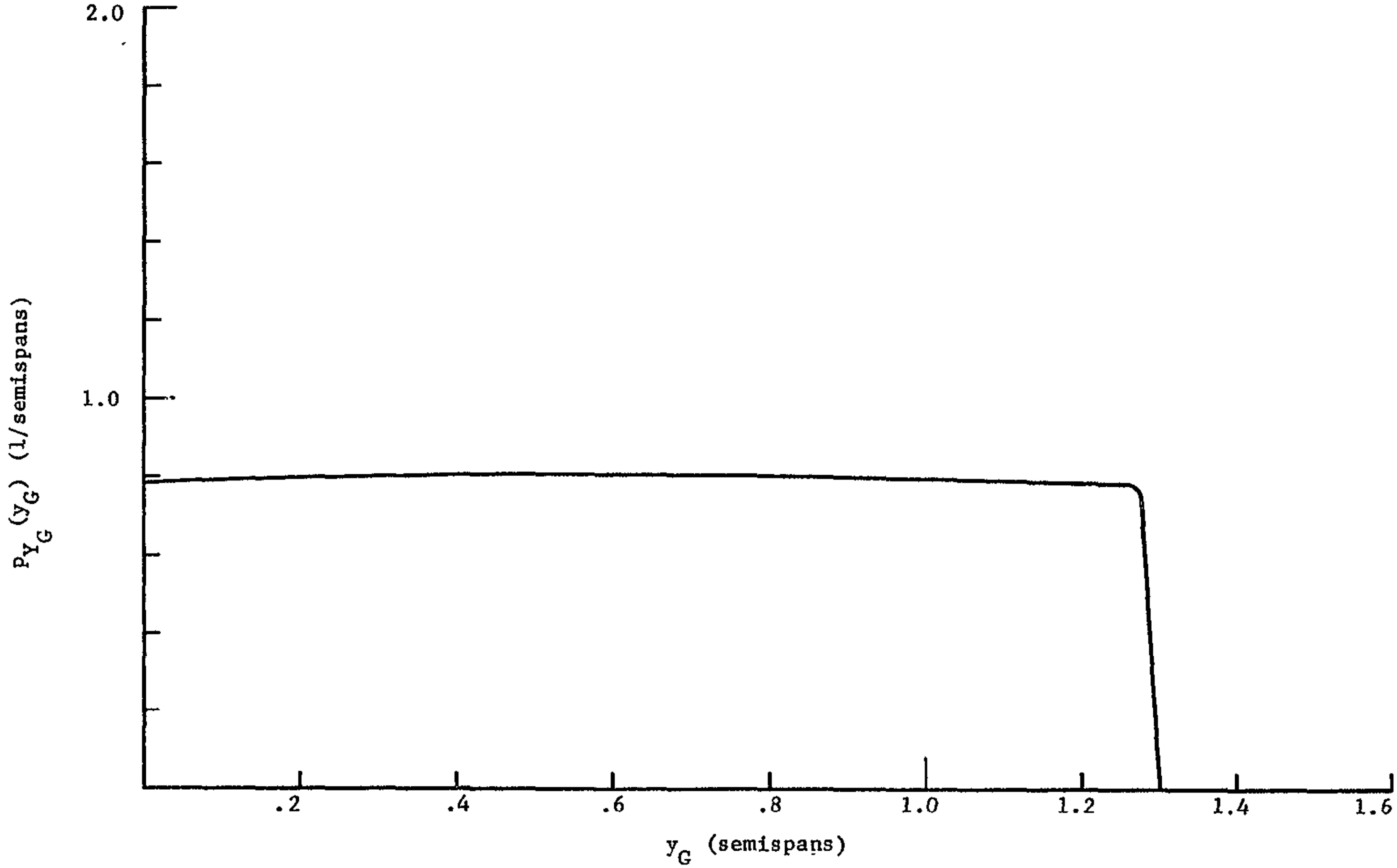

Figure 19. Corn, $0 \leq y_o \leq 0.2$

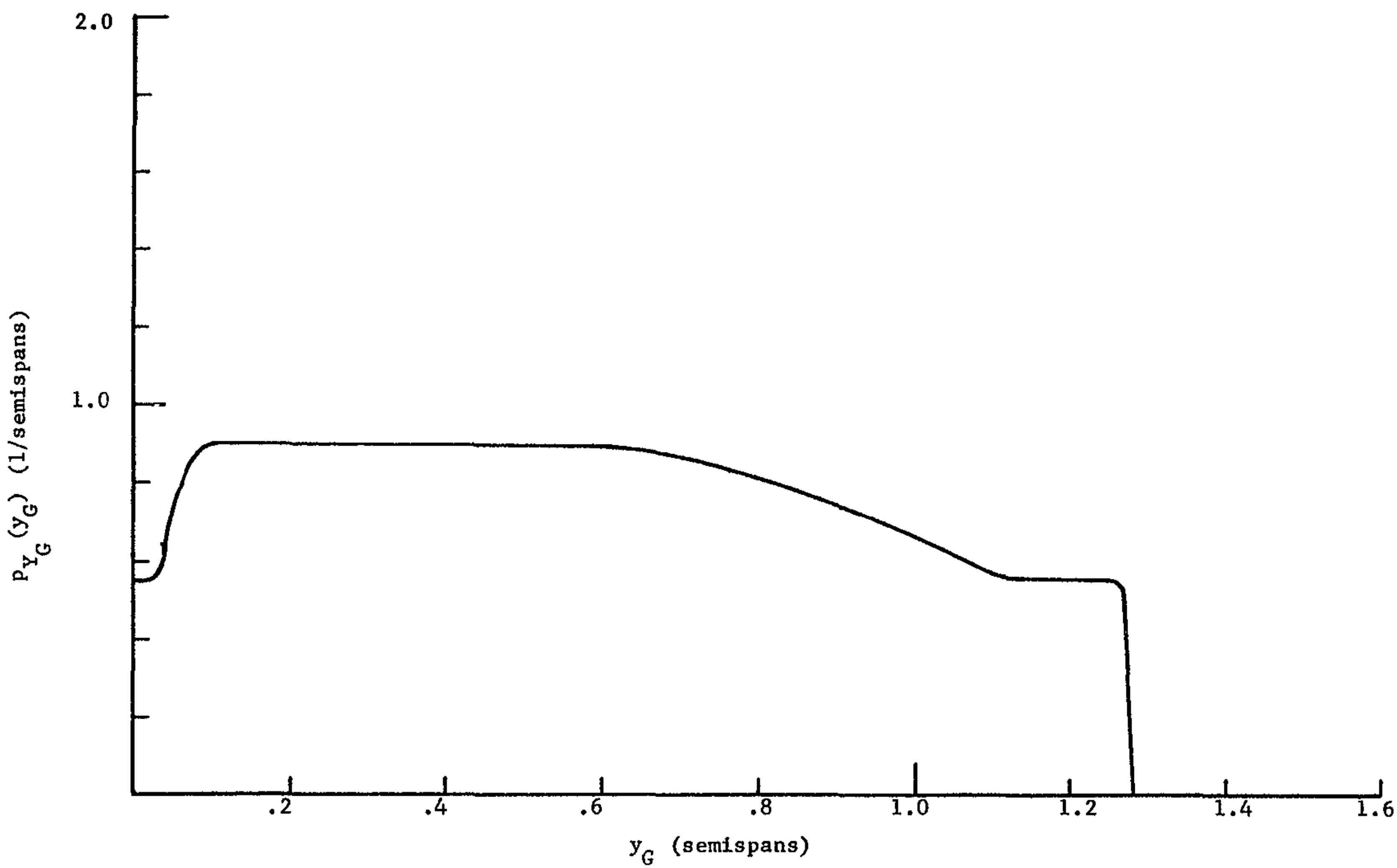

Figure 20. Oats, $0 \leq y_o \leq 0.2$

VI. SCALING CONSIDERATIONS

In order to facilitate experimental verification of these results and also to aid in obtaining viable results from test programs where scale model aircraft are used, it would be useful to investigate the effect of scaling the entire system on the probabilistic aspects of the problem. Specifically, it would be desirable to be able to predict probability density functions for the scaled values of the parameters in a given test which would result in distributions which are geometrically similar to those in a corresponding full scale system or a test of a different scale.

The problem of obtaining proper scaling for a deterministic system has been addressed by Ormsbee and Bragg (1978) and the derivation will not be presented here. Instead, a heuristic argument will be presented in order to gain insight into the nature of the probability problem, and then a formal approach will be undertaken.

Consider a collection of particles which are to be ejected from an aircraft. If two individual specimens are singled out, one possessing the mean diameter of the collection and the other possessing a diameter which is smaller by one standard deviation, and the trajectories of these particles are mapped, they would appear similar to those in Figure 21(a). In this argument, all other factors (i.e., the drag relation, the initial conditions and the density) are held to be deterministic. In order to obtain trajectories which are geometrically similar to each of these from a scale model test, it would be necessary to obtain two specimens from a collection of a different type of particle such that the requirements for scaling are met. These particles would describe trajectories similar to those shown in Figure 21(b).

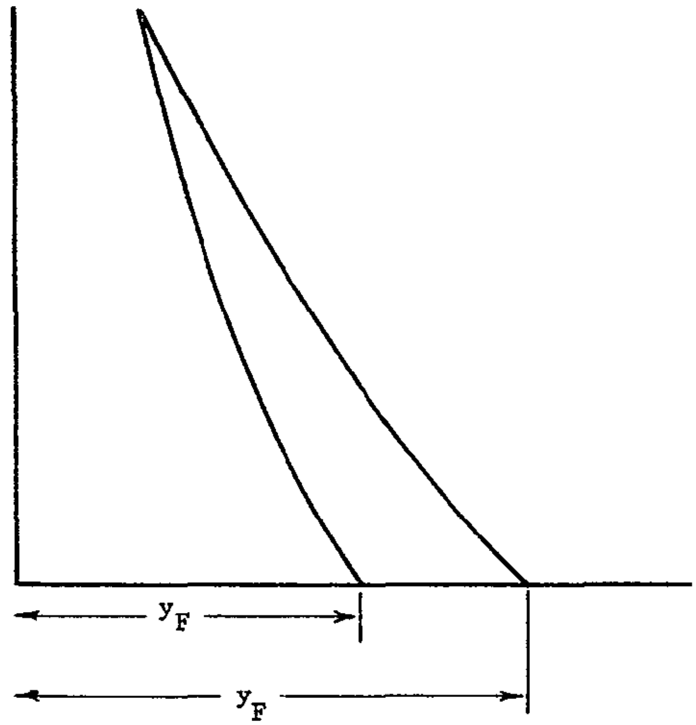

Figure 21(a). Full Scale

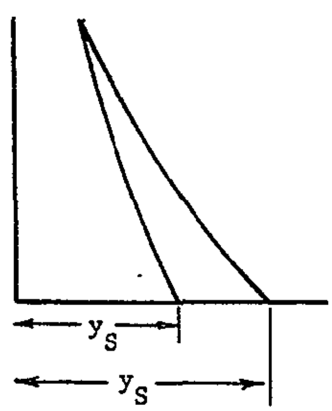

Figure 21(b). Scaled

It seems to follow that if the second collection of particles possesses mean and standard deviation values corresponding to the diameters of the specimens chosen, scaling of the distribution will have been achieved if the nondimensional lateral ground positions indicated in Figure 21 are identical. This presupposes that the shape of the probability density function has not changed so that it is sufficient only to insure that its width (a measure of which is the variance) has remained the same. This can be shown to be true if the scaling laws are observed.

More formally, proper scaling of the distribution will obtain if the probability density function for the modified ballistic parameter, α, remains identical. The only other possible inputs to the system are the geometry and the wake system and these have been accounted for by the deterministic scaling laws. A probability density function is completely determined by the values of its moments, the $i^{\underline{th}}$ moment about the mean being given by

$$\mu_i = \int_x (x-\mu_1)^i \; p_x(x)dx \; .$$

Since there are an infinite number of moments associated with a PDF, in general it is not possible to insure complete similarity. However, for the special case of the gaussian distribution (from which the lognormal is derived), all of the moments are not independent and, in fact, all the higher order moments can be written in terms of the first two: the mean and the variance. Thus the problem of distribution scaling is reduced to the question of maintaining the mean and standard deviation of the ballistic parameter.

In order to retain as much generality as possible, the earlier assumption that the density is deterministic will be removed and it will be assumed to have a lognormal PDF. Equation (28) can be rewritten to include this change as

$$\ln \alpha = \ln C' + \ln K - \ln \rho - 2 \ln \delta \tag{35}$$

where ρ is a nondimensional density obtained by dividing the particle density by the density of air. Letting

$$r = \ln \rho$$

and proceeding as before, the mean and variance of α can be determined, resulting in

$$\mu_\alpha = e^{\mu + \frac{\sigma^2}{2}} \tag{36}$$

$$\sigma_\alpha^2 = \mu_\alpha^2 (e^{\sigma^2} - 1) \tag{37}$$

where

$$\mu = \ln C' + \mu_y - \mu_r - 2\mu_x$$

$$\sigma^2 = \sigma_y^2 + \sigma_r^2 + 4\sigma_x^2$$

and

$$\sigma_y^2 = \ln[1 + (\frac{\sigma_K}{\mu_K})^2]$$

$$\sigma_x^2 = \ln[1 + (\frac{\sigma_\delta}{\mu_\delta})^2]$$

$$\sigma_r^2 = \ln[1 + (\frac{\sigma_\rho}{\mu_\rho})^2]$$

$$\mu_y = \ln \mu_K - \frac{\sigma_y^{\,2}}{2}$$

$$\mu_x = \ln \mu_\delta - \frac{\sigma_x^{\,2}}{2}$$

$$\mu_r = \ln \mu_\rho - \frac{\sigma_r^{\,2}}{2} \quad .$$

Thus scaling will be achieved if the values of the mean and variance as given by equations (36) and (37) are the same for the scaled system as for the full scale.

In the case of a general drag law, the deterministic values for the particle diameter and density necessary to insure scaling, if the aircraft is reduced in size by a factor of s, are as follows:

$$d_s = d_f \sqrt{s}$$

$$\rho_{p_s} = \rho_{p_f} \, s^{-\frac{3}{2}} \quad .$$

The corresponding nondimensional variables will have the following values:

$$\delta_s = \delta_f \, s^{\frac{3}{2}}$$

$$\rho_s = \rho_f \, s^{-\frac{3}{2}} \quad .$$

For this case it is necessary for the scaled particle to retain the same Reynolds number as the full scale and for the drag law to remain identical. The values required for the mean and variance of the scaled diameter and density which will insure scaling can be derived in the manner below. However, since both transformations are of the form

$$x_s = x_f \, s$$

it will suffice to carry through the derivation for just the diameter. First, observe that

$$\ln \delta_s = \ln \delta_f + \frac{3}{2} \ln s \ .$$

Since $\ln \delta_f$ is gaussian, this demonstrates that $\ln \delta_s$ must be gaussian and, consequently, δ_s must be lognormal. Letting

$$Z = \ln \delta_s$$

$$X = \ln \delta_f$$

$$C = \frac{3}{2} \ln s$$

and rewriting

$$Z = X + C \ .$$

The mean and variance of Z can be computed as before, giving

$$\mu_z = \mu_x + C$$

$$\sigma_z^{\ 2} = \sigma_x^{\ 2}$$

and, since δ_s is lognormal

$$\mu_{\delta_s} = e^{\mu_z + \frac{\sigma_z^{\ 2}}{2}}$$

$$= e^{\mu_x + C + \frac{\sigma_x^{\ 2}}{2}}$$

$$= e^C \, \mu_{\delta_f}$$

or

$$\mu_{\delta_s} = s^{3/2}\ \mu_{\delta_f} \ .$$

Likewise

$$\sigma_{\delta_s}{}^2 = \mu_{\delta_s}{}^2 (e^{\sigma_z{}^2} - 1)$$

$$= s^3\ \mu_{\delta_f}{}^2 (e^{\sigma_x{}^2} - 1)$$

$$= s^3\ \sigma_f{}^2$$

or

$$\sigma_{\delta_s} = s^{3/2}\ \sigma_{\delta_f} \ .$$

The derivation for the density proceeds identically, and results in

$$\mu_{\rho_s} = s^{-3/2}\ \mu_{\rho_f}$$

$$\sigma_{\rho_s} = s^{-3/2}\ \sigma_{\rho_f} \ .$$

Substitution back into equations (36) and (37) will verify the scaling since

$$C_s{}' = s^{3/2}\ C_f{}'$$

$$\sigma_s{}^2 = \sigma_f{}^2$$

$$\mu_s = \ln C' + \frac{3}{2} \ln s + \mu_y - \mu_{r_f} + \frac{3}{2} \ln s - 2\mu_{x_f} - 3 \ln s$$

$$= \mu_f \ .$$

If the analysis is restricted to the case where a simpler drag relation of the form

$$C_D = \frac{K}{Re}$$

is used (as in this study), a different result is obtained. As discussed by Ormsbee and Bragg, an approximation such as this frees the investigator from the separate requirements on density, diameter and drag relation and allows him instead to consider only their combination in the ballistic parameter. Similarly, in the probabilistic situation, it is not necessary to meet the requirements in the analysis immediately preceding, but only the more general ones defined by equations (36-37). Thus, as long as the PDF's for the density, diameter and drag relation are lognormal, any combination of means and variances for these parameters which satisfy equations (36-37) could be used in a scale test and still preserve the shape of the distribution.

VII. CONCLUSIONS

For granular particles which are larger than approximately 2500 microns in diameter, considerable simplification of the trajectory equations can be achieved due to the limited influence of the aircraft wake. The use of an approximate drag relation, which allowed further simplification, does not restrict the applicability as much as would at first appear, since numerical calculations show that the particle Reynolds number is maintained near the value associated with its terminal velocity for the entire time span of the trajectory, except for an initial acceleration which is of short duration. Thus the drag relation used need only approximate a small part of the drag curve in the range of the terminal Reynolds number.

In the case of smaller particles (below 2500 microns) the probability analysis could still be used if an empirical relation of the form

$$y_G = f(y_o, \alpha)$$

is used instead of the algebraic expressions derived here. It is only necessary to compute the Jacobian of the transformation, $\frac{dy_o}{dy_G}$, and to be able to write the above expression in a form

$$y_o = g(y_G, \alpha)$$

so that the distribution of y_G can be calculated.

The usefulness of this analysis can be demonstrated by noting that a distribution can be calculated by a single integration, whereas a compilation of trajectories would require one integration for each

trajectory. Moreover, if more than one initial condition is allowed to vary, the cost of producing a distribution by this method will increase by a factor equal to the number of initial conditions varied. On the other hand, the cost associated with a compilation of trajectories will increase by the power of the number used -- a geometric, rather than an arithmetic, increase.

It was shown that the shape of the distribution for a scale model test will be similar to that of a full scale test if, in the most general case, the mean values and variances of the particle parameters are scaled just as the deterministic values would be. Moreover, it was shown that if a drag relation of the form

$$C_D = \frac{K}{Re}$$

is used instead of a more general one, similarity could be maintained in any number of ways simply by observing a restriction on a combination of the means and variances of the particle parameters, rather than requiring separate concurrence with the scaling laws. This simplification offers a greater freedom in the choice of particles to be used in tests verifying these results and in any further work in which scale models are used. In view of the fact that particles meeting the requirements of the more general set of scaling laws have proved to be rather difficult to locate and/or implement, the simplified procedure for scaling offers a versatility which, hopefully, will more than offset any inaccuracies involved.

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APPENDIX

The computer code DEPOSIT was written to calculate the distributions by numerical integration using a FORTUOI catalog subroutine entitled GQU3Z. The input, where dimensions are required, can be done in any consistent system of units, due to the fact that all values are non-dimensionalized within the program, except the particle diameter, which must be input as the nondimensional δ, where

$$\delta = \frac{d}{b} .$$

A sample printout is shown, depicting the form of the output. By manipulating the variables RES and SC, the code can deliver any resolution of the distribution and can extend the range of calculation to cover any length of the lateral ground coordinate, y_G. Various types of distributor geometries can be analyzed by modification of the FUNCTION subprograms PYOF and PYOG. PYOF covers the approximation that

$$\alpha\tau \gg 1$$

while PYOF assumes that

$$\alpha \ll 1 .$$

```
      PROGRAM DEPOSIT(INPUT,OUTPUT)
      REAL NU,MD,MK,MA,MD1,MK1,MA1,KV
      EXTERNAL AUX
      COMMON/COM1/ YG,MA1,SA1,PI,MA
      COMMON/COM2/ ETA,XI,KV,WO,ZO,G,YI,YF
      READ2,MARK
      PRINT4,MARK
      PRINT5
      READ3,U,B,ZO,NU,RA
      PRINT6,U,B,ZO,NU,RA
      READ3,RP,MD,MK,SD,SK
      PRINT7,RP,MD,MK,SD,SK
      READ3,YI,YF,KV,WO SC
      PRINT8,YI,YF,KV,WO,SC
      READ3,UP,H,RES,ETA,XI
      PRINT80,UP,H,RES,ETA,XI
      PRINT9
      PI=3.14159265
      G=9.808*B/(U*U)
      C=0.75*NU*RA/(U*B*RP)
      SD1=SQRT(ALOG(1.0+(SD/MD)**2))
      SK1=SQRT(ALOG(1.0+(SK/MK)**2))
      MD1=ALOG(MD)-SD1*SD1/2
      MK1=ALOG(MK)-SK1*SK1/2
      PRINT10,C,SD1,SK1,MD1,MK1
      SA1=SQRT(4*SD1*SD1+SK1*SK1)
      MA1=ALOG(C)+MK1-2.*MD1
      MA=EXP(MA1+SA1*SA1/2 )
      SA=SQRT(MA*MA*(EXP(SA1*SA1)-1.))
      PRINT11,SA1,MA1,SA,MA
      PRINT12
      FYG=0.0E0
      NRES=IFIX(RES)+1
      DO1 I=1,NRES
      YG=(I-1)*SC/RES
      CALL GQU3Z(0.0E0,UP,AUX,H,PYG)
      FYG=FYG+PYG*SC/RES
      PRINT13,I,YG,PYG,FYG
    1 CONTINUE
      STOP
    2 FORMAT(A10)
    3 FORMAT(5E10.4)
    4 FORMAT("1",///10X,"GROUND DEPOSITION OF ",A10)
    5 FORMAT(//13X,"INPUT PARAMETERS")
    6 FORMAT(/"U =",F5.1,9X,"B =",F5.2,9X,"ZO=",F5.3,9X,"NU=",E12.6,2X,"
     &RA=",E12.6)
    7 FORMAT(/"RP=",E12.6,2X,"MD=",E12.6,2X,"MK=",E12 6 2X,"SD=",E12.6,2
     &X,"SK=",E12.6)
    8 FORMAT(/"YI=",F5.3,9X,"YF=",F5.3,9X,"KV=",F7.5,7X,"WO=",F8.5,6X,"S
     &C=",F5.3)
   80 FORMAT(/"UP=",F6.3,8X,"H =",F7.3,7X,"RE=",F7.3,7X,"ET=",E12.6,2X,"
```

```
    &XI=",E12.6)
  9  FORMAT(//,11X,"CALCULATED PARAMETERS")
 10  FORMAT(/" C =",E12.6,2X,"SD1=",E12.6,2X,"SK1=",E12.6,2X "MD1=",E12
    &.6,2X,"MK1=",E12.6)
 11  FORMAT(/"SA1=",E12.6,2X,"MA1=",E12.6,2X,"SA =",E12.6,2X,"MA =",E12
    &.6)
 12  FORMAT("1",///3X," I ",6X,"  YG ",9X,"PYG(YG)",8X,"FYG(YG)")
 13  FORMAT(3X,I3,3X,E12.6,3X,E12.6,3X,E12.6)
     END
     SUBROUTINE AUX(A,PY)
     REAL MA1,MA,KV
     COMMON/COM1/ YG,MA1,SA1,PI,MA
     COMMON/COM2/ ETA,XI,KV,WO,ZO G,YI,YF
     IF(A.LT.0.230) GOTO 1
     DYDG=1./(1.+KV/A)
     PYO=PYOF(YG,A)
     GO TO 2
  1  DYDG=1./(1.+KV*(WO+SQRT(WO*WO+2*ZO*G))/G)
     PYO=PYOG(YG,A)
  2  PEX=(ALOG(A)-MA1)**2/(2*SA1*SA1)
     IF(PEX GT.600.) GOTO 21
     PA=EXP(-PEX)/(SA1*A*SQRT(2*PI))
     GOTO 22
 21  PA=0.0E0
 22  CONTINUE
     PY=DYDG*PYO*PA
     RETURN
     END
     FUNCTION PYOF(YG,A)
     REAL KV
     COMMON/COM2/ ETA,XI,KV,WO ZO,G,YI,YF
     YO=(YG-ETA*(ZO*A+WO)/(A*(G/A-XI)))/(1.+KV/A)
     IF(YO.LT.YI) GOTO 1
     IF(YO.GT.YF) GOTO 1
     PYOF=1./(YF-YI)
     GOTO 2
  1  PYOF=0.0E0
  2  RETURN
     END
     FUNCTION PYOG(YG,A)
     REAL KV
     COMMON/COM2/ ETA,XI,KV,WO,ZO,G,YI,YF
     YO=YG/(1.+KV*(WO+SQRT(WO*WO+2*ZO*G))/G)
     IF(YO.LT.YI) GOTO 1
     IF(YO.GT.YF) GOTO 1
     PYOG=1./(YF-YI)
     GOTO 2
  1  PYOG=0.0E0
  2  RETURN
     END
```

1

GROUND DEPOSITION OF WHEAT

INPUT PARAMETERS

U = 30.5	B = 6.77	ZO= .500	NU= .146700E-04	RA= .122500E-02
RP= .132040E+01	MD= .528100E-03	MK= .127400E+04	SD= 279200E-04	SK= .226500E+03
YI=0.000	YF= 300	KV=1.00000	WO= .10000	SC=2.000
UP= 2.000	H = 48.000	RE=100.000	ET= .160000E-01	XI=- 180000E-01

CALCULATED PARAMETERS

C = .494673E-10	SD1= .528319E-01	SK1= .176405E+00	MD1=-.754762E+01	MK1= .713436E+01
SA1= .205630E+00	MA1=-.150011E+01	SA = .473571E-01	MA = .227873E+00	

1

I	YG	PYG(YG)	FYG(YG)
1	0.	.290713E+00	.581426E-02
2	.200000E-01	.290713E+00	.116285E-01
3	.400000E-01	.290713E+00	.174428E-01
4	.600000E-01	.600057E+00	.294439E-01
5	.800000E-01	.603464E+00	.415132E-01
6	.100000E+00	.603464E+00	.535825E-01
7	.120000E+00	.603464E+00	.656518E 01
8	.140000E+00	.603464E+00	.777210E-01
9	.160000E+00	.603464E+00	.897903E-01

26	.500000E+00	.603464E+00	.294968E+00
27	.520000E+00	.603464E+00	.307037E+00
28	.540000E+00	.603464E+00	.319107E+00
29	.560000E+00	.603464E+00	.331176E+00
30	.580000E+00	.603464E+00	.343245E+00
31	.600000E+00	.603464E+00	.355315E+00
32	.620000E+00	.603464E+00	.367384E+00
33	.640000E+00	.603464E+00	.379453E+00
34	.660000E+00	.603464E+00	.391522E+00
35	.680000E+00	.603464E+00	.403592E+00
36	.700000E+00	.603464E+00	.415661E+00
37	.720000E+00	.603464E+00	.427730E+00
38	.740000E+00	.603464E+00	.439800E+00
39	.760000E+00	.603464E+00	.451869E+00
40	.780000E+00	.603464E+00	.463938E+00
41	.800000E+00	.603464E+00	.476007E+00
42	.820000E+00	.603464E+00	.488077E+00
43	.840000E+00	.603464E+00	.500146E+00
44	.860000E+00	.603464E+00	.512215E+00
45	.880000E+00	.603463E+00	.524285E+00
46	.900000E+00	.603461E+00	.536354E+00
47	.920000E+00	.603457E+00	.548423E+00
48	.940000E+00	.603448E+00	.560492E+00
49	.960000E+00	.603429E+00	.572560E+00
50	.980000E+00	.603392E+00	.584628E+00
51	.100000E+01	.603329E+00	.596695E+00
52	.102000E+01	.603217E+00	.608759E+00
53	.104000E+01	.603032E+00	.620820E+00
54	.106000E+01	.602748E+00	.632875E+00
55	.108000E+01	.602319E+00	.644921E+00
56	.110000E+01	.601672E+00	.656955E+00
57	.112000E+01	.600782E+00	.668970E+00
58	.114000E+01	.599501E+00	.680960E+00
59	.116000E+01	.597838E+00	.692917E+00
60	.118000E+01	.595683E+00	.704831E+00
61	.120000E+01	.592904E+00	.716689E+00
62	.122000E+01	.589319E+00	.728475E+00
63	.124000E+01	.584761E+00	.740170E+00
64	.126000E+01	.579571E+00	.751762E+00
65	.128000E+01	.573285E+00	.763227E+00
66	.130000E+01	.566227E+00	.774552E+00
67	.132000E+01	.557848E+00	.785709E+00
68	.134000E+01	.548213E+00	.796673E+00
69	.136000E+01	.536765E+00	.807409E+00
70	.138000E+01	.525223E+00	.817913E+00
71	.140000E+01	.512360E+00	.828160E+00
72	.142000E+01	.498569E+00	.838132E+00
73	.144000E+01	.482121E+00	.847774E+00
74	.146000E+01	.467171E+00	.857117E+00
75	.148000E+01	.451056E+00	.866139E+00
76	.150000E+01	.432445E+00	.874787E+00

77	.152000E+01	.415287E+00	.883093E+00
78	.154000E+01	.396127E+00	.891016E+00
79	.156000E+01	.377620E+00	.898568E+00
80	.158000E+01	.358334E+00	.905735E+00
81	.160000E+01	.339880E+00	.912532E+00
82	.162000E+01	.318155E+00	.918895E+00
83	.164000E+01	.301217E+00	.924920E+00
84	.166000E+01	.290713E+00	.930734E+00
85	.168000E+01	.290713E+00	.936548E+00
86	.170000E+01	.290713E+00	.942363E+00
87	.172000E+01	.290713E+00	.948177E+00
88	.174000E+01	.290713E+00	.953991E+00
89	.176000E+01	.290713E+00	.959805E+00
90	.178000E+01	.290713E+00	.965620E+00
91	.180000E+01	.290713E+00	.971434E+00
92	.182000E+01	.290713E+00	.977248E+00
93	.184000E+01	.290713E+00	.983062E+00
94	.186000E+01	.290713E+00	.988877E+00
95	.188000E+01	.290713E+00	.994691E+00
96	.190000E+01	.290713E+00	.100051E+01
97	.192000E+01	0.	.100051E+01
98	.194000E+01	0.	.100051E+01
99	.196000E+01	0.	.100051E+01
100	.198000E+01	0.	.100051E+01
101	.200000E+01	0.	.100051E+01

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